MATHEMATICS

Textbook for Class VIII

0852

© NCERT not to be republished

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

Reprint 2024-25

0852 – MATHEMATICS
Textbook for Class VIII

ISBN 978-81-7450-814-0

First Edition
January 2008 Magha 1929

Reprinted
January 2009, January 2010, November 2010, January 2012, November 2012, November 2013, November 2014, December 2015, December 2016, December 2017, January 2019, August 2019 and January 2021

Revised Edition
November 2022 Kartika 1944

Reprinted
March 2024 Chaitra 1946

PD 800T SU

© National Council of Educational Research and Training, 2008, 2022

₹ 65.00

Printed on 80 GSM paper with NCERT watermark

Published at the Publication Division by the Secretary, National Council of Educational Research and Training, Sri Aurobindo Marg, New Delhi 110 016 and printed at Chandrakala Universal Pvt. Ltd., H-16, UPSIDC, Industrial Area, Newada, Samogar, Naini, Allahabad

ALL RIGHTS RESERVED

- No part of this publication may be reproduced, stored in a retrieval system or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise without the prior permission of the publisher.
- This book is sold subject to the condition that it shall not, by way of trade, be lent, re-sold, hired out or otherwise disposed of without the publisher's consent, in any form of binding or cover other than that in which it is published.
- The correct price of this publication is the price printed on this page, Any revised price indicated by a rubber stamp or by a sticker or by any other means is incorrect and should be unacceptable.

OFFICES OF THE PUBLICATION DIVISION, NCERT

NCERT Campus
Sri Aurobindo Marg
New Delhi 110 016 **Phone : 011-26562708**

108, 100 Feet Road
Hosdakere Halli Extension
Banashankari III Stage
Bengaluru 560 085 **Phone : 080-26725740**

Navjivan Trust Building
P.O. Navjivan
Ahmedabad 380 014 **Phone : 079-27541446**

CWC Campus
Opp. Dhankal Bus Stop
Panihati
Kolkata 700 114 **Phone : 033-25530454**

CWC Complex
Maligaon
Guwahati 781 021 **Phone : 0361-2674869**

Publication Team

Head, Publication Division	: *Anup Kumar Rajput*
Chief Editor	: *Shveta Uppal*
Chief Production Officer	: *Arun Chitkara*
Chief Business Manager (In charge)	: *Amitabh Kumar*
Production Officer	: *Atul Saxena*

Cover
Shweta Rao

Layout Design
Digital Expressions

Illustrations
Prashant Soni

© NCERT not to be republished

Foreword

The National Curriculum Framework, 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

NCERT appreciates the hard work done by the textbook development committee responsible for this book. We wish to thank the Chairperson of the advisory group in science and mathematics, Professor J.V. Narlikar and the Chief Advisor for this book, Dr H.K. Dewan for guiding the work of this committee. Several teachers contributed to the development of this textbook; we are grateful to their principals for making this possible. We are indebted to the institutions and organisations which have generously permitted us to draw upon their resources, material and personnel. As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director
National Council of Educational
Research and Training

New Delhi
30 November 2007

© NCERT not to be republished

© NCERT
not to be republished

Rationalisation of Content in the Textbooks

In view of the COVID-19 pandemic, it is imperative to reduce content load on students. The National Education Policy 2020, also emphasises reducing the content load and providing opportunities for experiential learning with creative mindset. In this background, the NCERT has undertaken the exercise to rationalise the textbooks across all classes. Learning Outcomes already developed by the NCERT across classes have been taken into consideration in this exercise.

Contents of the textbooks have been rationalised in view of the following:

- Overlapping with similar content included in other subject areas in the same class
- Similar content included in the lower or higher class in the same subject
- Difficulty level
- Content, which is easily accessible to students without much interventions from teachers and can be learned by children through self-learning or peer-learning
- Content, which is irrelevant in the present context

This present edition, is a reformatted version after carrying out the changes given above.

© NCERT not to be republished

© NCERT
not to be republished

Preface

This is the final book of the upper primary series. It has been an interesting journey to define mathematics learning in a different way. The attempt has been to retain the nature of mathematics, engage with the question why learn mathematics while making an attempt to create materials that would address the interest of the learners at this stage and provide sufficient and approachable challenge to them. There have been many views on the purpose of school mathematics. These range from the fully utilitarian to the entirely aesthetic perceptions. Both these end up not engaging with the concepts and enriching the apparatus available to the learner for participating in life. The NCF emphasises the need for developing the ability to mathematise ideas and perhaps experiences as well. An ability to explore the ideas and framework given by mathematics in the struggle to find a richer life and a more meaningful relationship with the world around.

This is not even easy to comprehend, far more difficult to operationalise. But NCF adds to this an even more difficult goal. The task is to involve everyone of that age group in the classroom or outside in doing mathematics. This is the aim we have been attempting to make in the series.

We have, therefore, provided space for children to engage in reflection, creating their own rules and definitions based on problems/tasks solved and following their ideas logically. The emphasis is not on remembering algorithms, doing complicated arithmetical problems or remembering proofs, but understanding how mathematics works and being able to identify the way of moving towards solving problems.

The important concern for us has also been to ensure that all students at this stage learn mathematics and begin to feel confident in relating mathematics. We have attempted to help children read the book and to stop and reflect at each step where a new idea has been presented. In order to make the book less formidable we have included illustrations and diagrams. These combined with the text help the child comprehend the idea. Throughout the series and also therefore in this book we have tried to avoid the use of technical words and complex formulations. We have left many things for the student to describe and write in her own words.

We have made an attempt to use child friendly language. To attract attention to some points blurbs have been used. The attempt has been to reduce the weight of long explanations by using these and the diagrams. The illustrations and fillers also attempt to break the monotony and provide contexts.

Class VIII is the bridge to Class IX where children will deal with more formal mathematics. The attempt here has been to introduce some ideas in a way that is moving towards becoming formal. The tasks included expect generalisation from the gradual use of such language by the child.

The team that developed this textbook consisted teachers with experience and appreciation of children learning mathematics. This team also included people with experience of research in mathematics teaching-learning and an experience of producing materials for children. The feedback on the textbooks for Classes VI and VII was kept in mind while developing this textbook. This process of development also included discussions with teachers during review workshop on the manuscript.

© NCERT not to be republished

In the end, I would like to express the grateful thanks of our team to Professor Krishna Kumar, *Director*, NCERT, Professor G. Ravindra, *Joint Director*, NCERT and Professor Hukum Singh, *Head*, DESM, for giving us an opportunity to work on this task with freedom and with full support. I am also grateful to Professor J.V. Narlikar, Chairperson of the Advisory Group in Science and Mathematics for his suggestions. I am also grateful for the support of the team members from NCERT, Professor S.K. Singh Gautam, Dr V.P. Singh and in particular Dr Ashutosh K. Wazalwar who coordinated this work and made arrangements possible. In the end I must thank the Publication Department of NCERT for its support and advice and those from Vidya Bhawan who helped produce the book.

It need not be said but I cannot help mentioning that all the authors worked as a team and we accepted ideas and advice from each other. We stretched ourselves to the fullest and hope that we have done some justice to the challenge posed before us.

The process of developing materials is, however, a continuous one and we would hope to make this book better. Suggestions and comments on the book are most welcome.

H.K. DEWAN
Chief Advisor
Textbook Development Committee

© NCERT not to be republished

A Note for the Teacher

This is the third and the last book of this series. It is a continuation of the processes initiated to help the learners in abstraction of ideas and principles of mathematics. Our students to be able to deal with mathematical ideas and use them need to have the logical foundations to abstract and use postulates and construct new formulations. The main points reflected in the NCF-2005 suggest relating mathematics to development of wider abilities in children, moving away from complex calculations and algorithm following to understanding and constructing a framework of understanding. As you know, mathematical ideas do not develop by telling them. They also do not reach children by merely giving explanations. Children need their own framework of concepts and a classroom where they are discussing ideas, looking for solutions to problems, setting new problems and finding their own ways of solving problems and their own definitions.

As we have said before, it is important to help children to learn to read the textbook and other books related to mathematics with understanding. The reading of materials is clearly required to help the child learn further mathematics. In Class VIII please take stock of where the students have reached and give them more opportunities to read texts that use language with symbols and have brevity and terseness with no redundancy. For this if you can, please get them to read other texts as well. You could also have them relate the physics they learn and the equations they come across in chemistry to the ideas they have learnt in mathematics. These cross-disciplinary references would help them develop a framework and purpose for mathematics. They need to be able to reconstruct logical arguments and appreciate the need for keeping certain factors and constraints while they relate them to other areas as well. Class VIII children need to have opportunity for all this.

As we have already emphasised, mathematics at the Upper Primary Stage has to be close to the experience and environment of the child and be abstract at the same time. From the comfort of context and/or models linked to their experience they need to move towards working with ideas. Learning to abstract helps formulate and understand arguments. The capacity to see interrelations among concepts helps us deal with ideas in other subjects as well. It also helps us understand and make better patterns, maps, appreciate area and volume and see similarities between shapes and sizes. While this is regarding the relationship of other fields of knowledge to mathematics, its meaning in life and our environment needs to be re-emphasised.

Children should be able to identify the principles to be used in contextual situations, for solving problems sift through and choose the relevant information as the first important step. Once students do that they need to be able to find the way to use the knowledge they have and reach where the problem requires them to go. They need to identify and define a problem, select or design possible solutions and revise or redesign the steps, if required. As they go further there would be more to of this to be done. In Class VIII we have to get them to be conscious of the steps they follow. Helping children to develop the ability to construct appropriate models by breaking up the problems and evolving their own strategies and analysis of problems is extremely important. This is in the place of giving them prescriptive algorithms.

© NCERT not to be republished

Cooperative learning, learning through conversations, desire and capacity to learn from each other and the recognition that conversation is not noise and consultation not cheating is an important part of change in attitude for you as a teacher and for the students as well. They should be asked to make presentations as a group with the inclusion of examples from the contexts of their own experiences. They should be encouraged to read the book in groups and formulate and express what they understand from it. The assessment pattern has to recognise and appreciate this and the classroom groups should be such that all children enjoy being with each other and are contributing to the learning of the group. As you would have seen different groups use different strategies. Some of these are not as efficient as others as they reflect the modeling done and reflect the thinking used. All these are appropriate and need to be analysed with children. The exposure to a variety of strategies deepens the mathematical understanding. Each group moves from where it is and needs to be given an opportunity for that.

For conciseness we present the key ideas of mathematics learning that we would like you to remember in your classroom.

1. Enquiry to understand is one of the natural ways by which students acquire and construct knowledge. The process can use generation of observations to acquire knowledge. Students need to deal with different forms of questioning and challenging investigations- explorative, open-ended, contextual and even error detection from geometry, arithmetic and generalising it to algebraic relations etc.
2. Children need to learn to provide and follow logical arguments, find loopholes in the arguments presented and understand the requirement of a proof. By now children have entered the formal stage. They need to be encouraged to exercise creativity and imagination and to communicate their mathematical reasoning both verbally and in writing.
3. The mathematics classroom should relate language to learning of mathematics. Children should talk about their ideas using their experiences and language. They should be encouraged to use their own words and language but also gradually shift to formal language and use of symbols.
4. The number system has been taken to the level of generalisation of rational numbers and their properties and developing a framework that includes all previous systems as sub-sets of the generalised rational numbers. Generalisations are to be presented in mathematical language and children have to see that algebra and its language helps us express a lot of text in small symbolic forms.
5. As before children should be required to set and solve a lot of problems. We hope that as the nature of the problems set up by them becomes varied and more complex, they would become confident of the ideas they are dealing with.
6. Class VIII book has attempted to bring together the different aspects of mathematics and emphasise the commonality. Unitary method, Ratio and proportion, Interest and dividends are all part of one common logical framework. The idea of variable and equations is needed wherever we need to find an unknown quantity in any branch of mathematics.

We hope that the book will help children learn to enjoy mathematics and be confident in the concepts introduced. We want to recommend the creation of opportunity for thinking individually and collectively.

We look forward to your comments and suggestions regarding the book and hope that you will send interesting exercises, activities and tasks that you develop during the course of teaching, to be included in the future editions. This can only happen if you would find time to listen carefully to children and identify gaps and on the other hand also find the places where they can be given space to articulate their ideas and verbalise their thoughts.

© NCERT not to be republished

Textbook Development Committee

Chairperson, Advisory Group in Science and Mathematics

J.V. Narlikar, Emeritus Professor, *Chairman,* Advisory Committee, Inter University Centre for Astronomy and Astrophysics (IUCCA), Ganeshkhind, Pune University, Pune

Chief Advisor

H.K. Dewan, Vidya Bhawan Society, Udaipur, Rajasthan

Chief Coordinator

Hukum Singh, *Professor and Head*, DESM, NCERT, New Delhi

Members

Anjali Gupte, *Teacher*, Vidya Bhawan Public School, Udaipur, Rajasthan

Avantika Dam, *TGT*, CIE Experimental Basic School, Department of Education, Delhi

B.C. Basti, *Senior Lecturer*, Regional Institute of Education, Mysore, Karnataka

H.C. Pradhan, *Professor*, Homi Bhabha Centre for Science Education, TIFR, Mumbai Maharashtra

K.A.S.S.V. Kameshwar Rao, *Lecturer*, Regional Institute of Education, Shyamala Hills Bhopal (M.P.)

Mahendra Shankar, *Lecturer (S.G.)* (Retd.), NCERT, New Delhi

Meena Shrimali, *Teacher*, Vidya Bhawan Senior Secondary School, Udaipur, Rajasthan

P. Bhaskar Kumar, *PGT*, Jawahar Navodaya Vidyalaya, Lepakshi, Distt. Anantpur (A.P.)

R. Athmaraman, *Mathematics Education Consultant*, TI Matric Higher Secondary School and AMTI, Chennai, Tamil Nadu

Ram Avtar, *Professor* (Retd.), NCERT, New Delhi

Shailesh Shirali, Rishi Valley School, Rishi Valley, Madanapalle (A.P.)

S.K.S. Gautam, *Professor*, DEME, NCERT, New Delhi

Shradha Agarwal, *Principal*, Florets International School, Panki, Kanpur (U.P.)

Srijata Das, *Senior Lecturer* in Mathematics, SCERT, New Delhi

V.P. Singh, *Reader*, DESM, NCERT, New Delhi

Member-coordinator

Ashutosh K. Wazalwar, *Professor*, DESM, NCERT, New Delhi

© NCERT not to be republished

ACKNOWLEDGEMENTS

The Council gratefully acknowledges the valuable contributions of the following participants of the Textbook Review Workshop: Shri Pradeep Bhardwaj, *TGT* (Mathematics) Bal Sthali Public Secondary School, Kirari, Nangloi, New Delhi; Shri Sankar Misra, *Teacher* in Mathematics, Demonstration Multipurpose School, Regional Institute of Education, Bhubaneswar (Orissa); Shri Manohar M. Dhok, *Supervisor,* M.P. Deo Smruti Lokanchi Shala, Nagpur (Maharashtra); Shri Manjit Singh Jangra, *Maths teacher,* Government Senior Secondary School, Sector-4/7, Gurgoan (Haryana); Dr. Rajendra Kumar Pooniwala, U.D.T., Government Subhash Excellence School, Burhanpur (M.P.); Shri K. Balaji, TGT (Mathematics), Kendriya Vidyalaya No.1, Tirupati (A.P.); Ms. Mala Mani, Amity International School, Sector-44, Noida; Ms. Omlata Singh, *TGT* (Mathematics), Presentation Convent Senior Secondary School, Delhi; Ms. Manju Dutta, Army Public School, Dhaula Kuan, New Delhi; Ms. Nirupama Sahni, *TGT* (Mathematics), Shri Mahaveer Digambar Jain Senior Secondary School, Jaipur (Rajasthan); Shri Nagesh Shankar Mone, *Head Master,* Kantilal Purshottam Das Shah Prashala, Vishrambag, Sangli (Maharashtra); Shri Anil Bhaskar Joshi, *Senior teacher* (Mathematics), Manutai Kanya Shala, Tilak Road, Akola (Maharashtra); Dr. Sushma Jairath, *Reader,* DWS, NCERT, New Delhi; Shri Ishwar Chandra, *Lecturer (S.G.)* (Retd.) NCERT, New Delhi.

The Council is grateful for the suggestions/comments given by the following participants during the workshop of the mathematics Textbook Development Committee – Shri Sanjay Bolia and Shri Deepak Mantri from Vidya Bhawan Basic School, Udaipur; Shri Inder Mohan Singh Chhabra, Vidya Bhawan Educational Resource Centre, Udaipur.

The Council acknowledges the comments/suggestions given by Dr. R.P. Maurya, *Reader,* DESM, NCERT, New Delhi; Dr. Sanjay Mudgal, *Lecturer,* DESM, NCERT, New Delhi; Dr. T.P. Sharma, *Lecturer,* DESM, NCERT, New Delhi for the improvement of the book.

The Council acknowledges the support and facilities provided by Vidya Bhawan Society and its staff, Udaipur, for conducting workshops of the development committee at Udaipur and to the Director, Centre for Science Education and Communication (CSEC), Delhi University for providing library help.

The Council acknowledges the academic and administrative support of Professor Hukum Singh, Head, DESM, NCERT.

The Council also acknowledges the efforts of Sajjad Haider Ansari, Rakesh Kumar, Neelam Walecha, *DTP Operators*; Kanwar Singh, *Copy Editor;* Abhimanu Mohanty, *Proof Reader*, Deepak Kapoor, *Computer Station Incharge*, DESM, NCERT for technical assistance, APC Office and the Administrative Staff, DESM, NCERT and the Publication Department of the NCERT.

The book was reviewed by Rahul Sofat, *PGT* Mathematics, Air Force Golden Jubilee School, Subroto Park, New Delhi; Gurpreet Bhatnagar, CBSE Resource Person; A. K. Wazalwar and Til Prasad Sarma, DESM, NCERT, New Delhi.

© NCERT not to be republished

Contents

	Foreword	*iii*
	Rationalised Content in the Textbooks	*v*
	Preface	*vii*
Chapter 1	Rational Numbers	1
Chapter 2	Linear Equations in One Variable	15
Chapter 3	Understanding Quadrilaterals	21
Chapter 4	Data Handling	37
Chapter 5	Squares and Square Roots	51
Chapter 6	Cubes and Cube Roots	71
Chapter 7	Comparing Quantities	79
Chapter 8	Algebraic Expressions and Identities	93
Chapter 9	Mensuration	103
Chapter 10	Exponents and Powers	121
Chapter 11	Direct and Inverse Proportions	129
Chapter 12	Factorisation	145
Chapter 13	Introduction to Graphs	157
	Answers	171
	Just for Fun	180

© NCERT not to be republished

Constitution of India

Part IV A (Article 51 A)

Fundamental Duties

It shall be the duty of every citizen of India —

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wildlife and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

*(k) who is a parent or guardian, to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

Note: The Article 51A containing Fundamental Duties was inserted by the Constitution (42nd Amendment) Act, 1976 (with effect from 3 January 1977).

*(k) was inserted by the Constitution (86th Amendment) Act, 2002 (with effect from 1 April 2010).

© NCERT not to be republished

ANSWERS

EXERCISE 1.1

1. (i) 1 is the multiplicative identity (ii) Commutativity
(iii) Multiplicative inverse

2. Rational number

EXERCISE 2.1

1. $x = 18$ **2.** $t = -1$ **3.** $x = -2$ **4.** $z = \frac{3}{2}$ **5.** $x = 5$ **6.** $x = 0$

7. $x = 40$ **8.** $x = 10$ **9.** $y = \frac{7}{3}$ **10.** $m = \frac{4}{5}$

EXERCISE 2.2

1. $x = \frac{27}{10}$ **2.** $n = 36$ **3.** $x = -5$ **4.** $x = 8$ **5.** $t = 2$

6. $m = \frac{7}{5}$ **7.** $t = -2$ **8.** $y = \frac{2}{3}$ **9.** $z = 2$ **10.** $f = 0.6$

EXERCISE 3.1

1. (a) 1, 2, 5, 6, 7 (b) 1, 2, 5, 6, 7 (c) 1, 2
(d) 2 (e) 1

2. A polygon with equal sides and equal angles.
(i) Equilateral triangle (ii) Square (iii) Regular hexagon

EXERCISE 3.2

1. (a) $360° - 250° = 110°$ (b) $360° - 310° = 50°$

2. (i) $\frac{360°}{9} = 40°$ (ii) $\frac{360°}{15} = 24°$

3. $\frac{360}{24} = 15$ (sides) **4.** Number of sides = 24

5. (a) No; (Since 22 is not a divisor of 360)
(b) No; (because each exterior angle is $180° - 22° = 158°$, which is not a divisor of 360°).

6. (a) The equilateral triangle being a regular polygon of 3 sides has the least measure of an interior angle = 60°.
(b) By (a), we can see that the greatest exterior angle is 120°.

© NCERT not to be republished

EXERCISE 3.3

1. (i) BC(Opposite sides are equal) (ii) $\angle$DAB (Opposite angles are equal)
(iii) OA (Diagonals bisect each other)
(iv) 180° (Interior opposite angles, since $\overline{AB} \parallel \overline{DC}$)

2. (i) $x = 80°; y = 100°; z = 80°$ (ii) $x = 130°; y = 130°; z = 130°$
(iii) $x = 90°; y = 60°; z = 60°$ (iv) $x = 100°; y = 80°; z = 80°$
(v) $y = 112°; x = 28°; z = 28°$

3. (i) Can be, but need not be.
(ii) No; (in a parallelogram, opposite sides are equal; but here, AD $\neq$ BC).
(iii) No; (in a parallelogram, opposite angles are equal; but here, $\angle A \neq \angle C$).

4. A kite, for example **5.** 108°; 72°; **6.** Each is a right angle.

7. $x = 110°; y = 40°; z = 30°$

8. (i) $x = 6; y = 9$ (ii) $x = 3; y = 13;$ **9.** $x = 50°$

10. $\overline{NM} \parallel \overline{KL}$ (sum of interior opposite angles is 180°). So, KLMN is a trapezium.

11. 60° **12.** $\angle P = 50°; \angle S = 90°$

EXERCISE 3.4

1. (b), (c), (f), (g), (h) are true; others are false.

2. (a) Rhombus; square. (b) Square; rectangle

3. (i) A square is 4 – sided; so it is a quadrilateral.
(ii) A square has its opposite sides parallel; so it is a parallelogram.
(iii) A square is a parallelogram with all the 4 sides equal; so it is a rhombus.
(iv) A square is a parallelogram with each angle a right angle; so it is a rectangle.

4. (i) Parallelogram; rhombus; square; rectangle.
(ii) Rhombus; square (iii) Square; rectangle

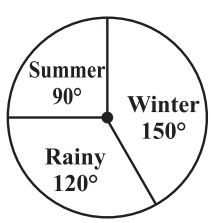
5. Both of its diagonals lie in its interior.

6. $\overline{AD} \parallel \overline{BC}$; $\overline{AB} \parallel \overline{DC}$. So, in parallelogram ABCD, the mid-point of diagonal $\overline{AC}$ is O.

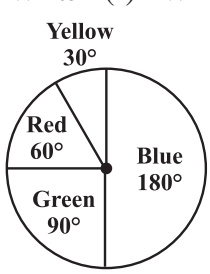
EXERCISE 4.1

1. (i) 200 (ii) Light music (iii) Classical - 100, Semi classical - 200, Light - 400, Folk - 300

2. (i) Winter (ii) Winter - 150°, Rainy - 120°, Summer - 90° (iii)

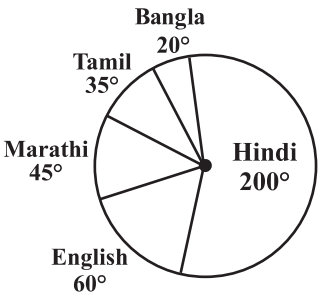


3.



© NCERT not to be republished

4. (i) Hindi (ii) 30 marks (iii) Yes **5.**



EXERCISE 4.2

1. (a) Outcomes → A, B, C, D

(b) HT, HH, TH, TT (Here HT means Head on first coin and Tail on the second coin and so on).

2. Outcomes of an event of getting

(i) (a) 2, 3, 5 (b) 1, 4, 6

(ii) (a) 6 (b) 1, 2, 3, 4, 5

3. (a) $\frac{1}{5}$ (b) $\frac{1}{13}$ (c) $\frac{4}{7}$

4. (i) $\frac{1}{10}$ (ii) $\frac{1}{2}$ (iii) $\frac{2}{5}$ (iv) $\frac{9}{10}$

5. Probability of getting a green sector = $\frac{3}{5}$; probability of getting a non-blue sector = $\frac{4}{5}$

6. Probability of getting a prime number = $\frac{1}{2}$; probability of getting a number which is not prime = $\frac{1}{2}$

Probability of getting a number greater than 5 = $\frac{1}{6}$

Probability of getting a number not greater than 5 = $\frac{5}{6}$

EXERCISE 5.1

1. (i) 1 (ii) 4 (iii) 1 (iv) 9 (v) 6 (vi) 9

(vii) 4 (viii) 0 (ix) 6 (x) 5

2. These numbers end with

(i) 7 (ii) 3 (iii) 8 (iv) 2 (v) 0 (vi) 2

(vii) 0 (viii) 0

3. (i), (iii) **4.** 10000200001, 100000020000001

5. 1020304030201, 101010101^2 **6.** 20, 6, 42, 43

7. (i) 25 (ii) 100 (iii) 144

8. (i) 1 + 3 + 5 + 7 + 9 + 11 + 13

(ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21

9. (i) 24 (ii) 50 (iii) 198

© NCERT not to be republished

EXERCISE 5.2

1. (i) 1024 (ii) 1225 (iii) 7396 (iv) 8649 (v) 5041 (vi) 2116

2. (i) 6,8,10 (ii) 14,48,50 (iii) 16,63,65 (iv) 18,80,82

EXERCISE 5.3

1. (i) 1, 9 (ii) 4, 6 (iii) 1, 9 (iv) 5

2. (i), (ii), (iii) **3.** 10, 13

4. (i) 27 (ii) 20 (iii) 42 (iv) 64 (v) 88 (vi) 98
(vii) 77 (viii) 96 (ix) 23 (x) 90

5. (i) 7; 42 (ii) 5; 30 (iii) 7, 84 (iv) 3; 78 (v) 2; 54 (vi) 3; 48

6. (i) 7; 6 (ii) 13; 15 (iii) 11; 6 (vi) 5; 23 (v) 7; 20 (vi) 5; 18

7. 49 **8.** 45 rows; 45 plants in each row **9.** 900 **10.** 3600

EXERCISE 5.4

1. (i) 48 (ii) 67 (iii) 59 (iv) 23 (v) 57 (vi) 37
(vii) 76 (viii) 89 (ix) 24 (x) 32 (xi) 56 (xii) 30

2. (i) 1 (ii) 2 (iii) 2 (iv) 3 (v) 3

3. (i) 1.6 (ii) 2.7 (iii) 7.2 (iv) 6.5 (v) 5.6

4. (i) 2; 20 (ii) 53; 44 (iii) 1; 57 (iv) 41; 28 (v) 31; 63

5. (i) 4; 23 (ii) 14; 42 (iii) 4; 16 (iv) 24; 43 (v) 149; 81

6. 21 m **7.** (a) 10 cm (b) 12 cm

8. 24 plants **9.** 16 children

EXERCISE 6.1

1. (ii) and (iv)

2. (i) 3 (ii) 2 (iii) 3 (iv) 5 (v) 10

3. (i) 3 (ii) 2 (iii) 5 (iv) 3 (v) 11

4. 20 cuboids

EXERCISE 6.2

1. (i) 4 (ii) 8 (iii) 22 (iv) 30 (v) 25 (vi) 24
(vii) 48 (viii) 36 (ix) 56

2. (i) False (ii) True (iii) False (iv) False (v) False (vi) False
(vii) True

© NCERT not to be republished

EXERCISE 7.1

1. (a) 1 : 2 (b) 1 : 2000 (c) 1 : 10

2. (a) 75% (b) $66\frac{2}{3}\%$ **3.** 28% students **4.** 25 matches **5.** ₹ 2400

6. 10%, cricket → 30 lakh; football → 15 lakh; other games → 5 lakh

EXERCISE 7.2

1. ₹ 2,835 **2.** ₹ 14,560

3. ₹ 2,000 **4.** ₹ 5,000 **5.** ₹ 1,050

EXERCISE 7.3

1. (i) About 48,980 (ii) 59,535 **2.** 5,31,616 (approx)

3. ₹ 38,640

EXERCISE 8.1

1. (i) 0 (ii) $ab + bc + ac$ (iii) $-p^2q^2 + 4pq + 9$

(iv) $2(l^2 + m^2 + n^2 + lm + mn + nl)$

2. (a) $8a - 2ab + 2b - 15$ (b) $2xy - 7yz + 5zx + 10xyz$

(c) $p^2q - 7pq^2 + 8pq - 18q + 5p + 28$

EXERCISE 8.2

1. (i) $28p$ (ii) $-28p^2$ (iii) $-28p^2q$ (iv) $-12p^4$ (v) 0

2. pq; $50\,mn$; $100\,x^2y^2$; $12x^3$; $12mn^2p$

3.

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

© NCERT not to be republished

4. (i) $105a^7$ (ii) $64pqr$ (iii) $4x^4y^4$ (iv) $6abc$

5. (i) $x^2y^2z^2$ (ii) $-a^6$ (iii) $1024y^6$ (iv) $36a^2b^2c^2$ (v) $-m^3n^2p$

EXERCISE 8.3

1. (i) $4pq + 4pr$ (ii) $a^2b - ab^2$ (iii) $7a^3b^2 + 7a^2b^3$
(iv) $4a^3 - 36a$ (v) 0

2. (i) $ab + ac + ad$ (ii) $5x^2y + 5xy^2 - 25xy$
(iii) $6p^3 - 7p^2 + 5p$ (iv) $4p^4q^2 - 4p^2q^4$
(v) $a^2bc + ab^2c + abc^2$

3. (i) $8a^{50}$ (ii) $-\frac{3}{5}x^3y^3$ (iii) $-4p^4q^4$ (iv) x^{10}

4. (a) $12x^2 - 15x + 3$; (i) 66 (ii) $\frac{-3}{2}$
(b) $a^3 + a^2 + a + 5$; (i) 5 (ii) 8 (iii) 4

5. (a) $p^2 + q^2 + r^2 - pq - qr - pr$ (b) $-2x^2 - 2y^2 - 4xy + 2yz + 2zx$
(c) $5l^2 + 25ln$ (d) $-3a^2 - 2b^2 + 4c^2 - ab + 6bc - 7ac$

EXERCISE 8.4

1. (i) $8x^2 + 14x - 15$ (ii) $3y^2 - 28y + 32$ (iii) $6.25l^2 - 0.25m^2$
(iv) $ax + 5a + 3bx + 15b$ (v) $6p^2q^2 + 5pq^3 - 6q^4$ (vi) $3a^4 + 10a^2b^2 - 8b^4$

2. (i) $15 - x - 2x^2$ (ii) $7x^2 + 48xy - 7y^2$ (iii) $a^3 + a^2b^2 + ab + b^3$
(iv) $2p^3 + p^2q - 2pq^2 - q^3$

3. (i) $x^3 + 5x^2 - 5x$ (ii) $a^2b^3 + 3a^2 + 5b^3 + 20$ (iii) $t^3 - st + s^2t^2 - s^3$
(iv) $4ac$ (v) $3x^2 + 4xy - y^2$ (vi) $x^3 + y^3$
(vii) $2.25x^2 - 16y^2$ (viii) $a^2 + b^2 - c^2 + 2ab$

EXERCISE 9.1

1. 0.88 m^2 **2.** 7 cm **3.** 660 m^2 **4.** 252 m^2

5. 45 cm^2 **6.** 24 cm^2, 6 cm **7.** ₹ 810 **8.** 140 m

9. 119 m^2 **10.** Area using Jyoti's way $= 2 \times \frac{1}{2} \times \frac{15}{2} \times (30 + 15)\ m^2 = 337.5\ m^2$,

Area using Kavita's way $= \frac{1}{2} \times 15 \times 15 + 15 \times 15 = 337.5\ m^2$

11. 80 cm^2, 96 cm^2, 80 cm^2, 96 cm^2

EXERCISE 9.2

1. (a) **2.** 144 m **3.** 10 cm **4.** 11 m^2

5. 5 cans

6. Similarity → Both have same heights. Difference → one is a cylinder, the other is a cube. The cube has larger lateral surface area

7. 440 m^2 **8.** 322 cm **9.** 1980 m^2 **10.** 704 cm^2

© NCERT not to be republished

EXERCISE 9.3

1. (a) Volume (b) Surface area (c) Volume

2. Volume of cylinder B is greater; Surface area of cylinder B is greater.

3. 5 cm **4.** 450 **5.** 1 m **6.** 49500 L

7. (i) 4 times (ii) 8 times **8.** 30 hours

EXERCISE 10.1

1. (i) $\frac{1}{9}$ (ii) $\frac{1}{16}$ (iii) 32

2. (i) $\frac{1}{(-4)^3}$ (ii) $\frac{1}{2^6}$ (iii) $(5)^4$ (iv) $\frac{1}{(3)^2}$ (v) $\frac{1}{(-14)^3}$

3. (i) 5 (ii) $\frac{1}{2}$ (iii) 29 (iv) 1 (v) $\frac{81}{16}$

4. (i) 250 (ii) $\frac{1}{60}$ **5.** $m = 2$ **6.** (i) -1 (ii) $\frac{512}{125}$

7. (i) $\frac{625t^4}{2}$ (ii) 5^5

EXERCISE 10.2

1. (i) 8.5×10^{-12} (ii) 9.42×10^{-12} (iii) 6.02×10^{15}
(iv) 8.37×10^{-9} (v) 3.186×10^{10}

2. (i) 0.00000302 (ii) 45000 (iii) 0.00000003
(iv) 1000100000 (v) 5800000000000 (vi) 3614920

3. (i) 1×10^{-6} (ii) 1.6×10^{-19} (iii) 5×10^{-7}
(iv) 1.275×10^{-5} (v) 7×10^{-2}

4. 1.0008×10^2

EXERCISE 11.1

1. No

2.

Parts of red pigment	1	4	7	12	20
Parts of base	8	32	56	96	160

3. 24 parts **4.** 700 bottles **5.** 10^{-4} cm; 2 cm **6.** 21 m

7. (i) 2.25×10^7 crystals (ii) 5.4×10^6 crystals **8.** 4 cm

9. (i) 6 m (ii) 8 m 75 cm **10.** 168 km

© NCERT not to be republished

EXERCISE 11.2

1. (i), (iv), (v) **2.** $4 \to 25{,}000$; $5 \to 20{,}000$; $8 \to 12{,}500$; $10 \to 10{,}000$; $20 \to 5{,}000$
Amount given to a winner is inversely proportional to the number of winners.

3. $8 \to 45°$, $10 \to 36°$, $12 \to 30°$ (i) Yes (ii) 24° (iii) 9

4. 6 **5.** 4 **6.** 3 days **7.** 15 boxes

8. 49 machines **9.** $1\frac{1}{2}$ hours **10.** (i) 6 days (ii) 6 persons **11.** 40 minutes

EXERCISE 12.1

1. (i) 12 (ii) $2y$ (iii) $14pq$ (iv) 1 (v) $6ab$ (vi) $4x$
(vii) 10 (viii) x^2y^2

2. (i) $7(x-6)$ (ii) $6(p-2q)$ (iii) $7a(a+2)$ (iv) $4z(-4+5z^2)$
(v) $10\ lm(2l+3a)$ (vi) $5xy(x-3y)$ (vii) $5(2a^2-3b^2+4c^2)$
(viii) $4a(-a+b-c)$ (ix) $xyz(x+y+z)$ (x) $xy(ax+by+cz)$

3. (i) $(x+8)\,(x+y)$ (ii) $(3x+1)\,(5y-2)$ (iii) $(a+b)\,(x-y)$
(iv) $(5p+3)\,(3q+5)$ (v) $(z-7)\,(1-xy)$

EXERCISE 12.2

1. (i) $(a+4)^2$ (ii) $(p-5)^2$ (iii) $(5m+3)^2$ (iv) $(7y+6z)^2$
(v) $4(x-1)^2$ (vi) $(11b-4c)^2$ (vii) $(l-m)^2$ (viii) $(a^2+b^2)^2$

2. (i) $(2p-3q)\,(2p+3q)$ (ii) $7(3a-4b)\,(3a+4b)$ (iii) $(7x-6)\,(7x+6)$
(iv) $16x^3(x-3)\,(x+3)$ (v) $4lm$ (vi) $(3xy-4)\,(3xy+4)$
(vii) $(x-y-z)\,(x-y+z)$ (viii) $(5a-2b+7c)\,(5a+2b-7c)$

3. (i) $x(ax+b)$ (ii) $7(p^2+3q^2)$ (iii) $2x(x^2+y^2+z^2)$
(iv) $(m^2+n^2)\,(a+b)$ (v) $(l+1)\,(m+1)$ (vi) $(y+9)\,(y+z)$
(vii) $(5y+2z)\,(y-4)$ (viii) $(2a+1)\,(5b+2)$ (ix) $(3x-2)\,(2y-3)$

4. (i) $(a-b)\,(a+b)\,(a^2+b^2)$ (ii) $(p-3)\,(p+3)\,(p^2+9)$
(iii) $(x-y-z)\,(x+y+z)\,[x^2+(y+z)^2]$ (iv) $z(2x-z)\,(2x^2-2xz+z^2)$
(v) $(a-b)^2\,(a+b)^2$

5. (i) $(p+2)\,(p+4)$ (ii) $(q-3)\,(q-7)$ (iii) $(p+8)\,(p-2)$

EXERCISE 12.3

1. (i) $\frac{x^3}{2}$ (ii) $-4y$ (iii) $6pqr$ (iv) $\frac{2}{3}x^2y$ (v) $-2a^2b^4$

2. (i) $\frac{1}{3}(5x-6)$ (ii) $3y^4-4y^2+5$ (iii) $2(x+y+z)$

(iv) $\frac{1}{2}(x^2+2x+3)$ (v) q^3-p^3

© NCERT not to be republished

3. (i) $2x - 5$ (ii) 5 (iii) $6y$ (iv) xy (v) $10abc$

4. (i) $5(3x + 5)$ (ii) $2y(x + 5)$ (iii) $\frac{1}{2}r(p + q)$ (iv) $4(y^2 + 5y + 3)$
(v) $(x + 2)(x + 3)$

5. (i) $y + 2$ (ii) $m - 16$ (iii) $5(p - 4)$ (iv) $2z(z - 2)$ (v) $\frac{5}{2}q(p - q)$
(vi) $3(3x - 4y)$ (vii) $3y(5y - 7)$

EXERCISE 13.1

1. (a) 36.5° C (b) 12 noon (c) 1 p.m, 2 p.m.
(d) 36.5° C; The point between 1 p.m. and 2 p.m. on the x-axis is equidistant from the two points showing 1 p.m. and 2 p.m., so it will represent 1.30 p.m. Similarly, the point on the y-axis, between 36° C and 37° C will represent 36.5° C.
(e) 9 a.m. to 10 a.m., 10 a.m. to 11 a.m., 2 p.m. to 3 p.m.

2. (a) (i) ₹ 4 crore (ii) ₹ 8 crore
(b) (i) ₹ 7 crore (ii) ₹ 8.5 crore (approx.)
(c) ₹ 4 crore (d) 2005

3. (a) (i) 7 cm (ii) 9 cm
(b) (i) 7 cm (ii) 10 cm
(c) 2 cm (d) 3 cm (e) Second week (f) First week
(g) At the end of the 2nd week

4. (a) Tue, Fri, Sun (b) 35° C (c) 15° C (d) Thurs

6. (a) 4 units = 1 hour (b) $3\frac{1}{2}$ hours (c) 22 km
(d) Yes; This is indicated by the horizontal part of the graph (10 a.m. - 10.30 a.m.)
(e) Between 8 a.m. and 9 a.m.

7. (iii) is not possible

EXERCISE 13.2

1. (b) (i) 20 km (ii) 7.30 a.m. (c) (i) Yes (ii) ₹ 200 (iii) ₹ 3500

2. (i) Yes (ii) No

© NCERT not to be republished

JUST FOR FUN

1. More about Pythagorean triplets

We have seen one way of writing pythagorean triplets as $2m, m^2 - 1, m^2 + 1$.

A pythagorean triplet a, b, c means $a^2 + b^2 = c^2$. If we use two natural numbers m and $n (m > n)$, and take $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$, then we can see that $c^2 = a^2 + b^2$.

Thus for different values of m and n with $m > n$ we can generate natural numbers a, b, c such that they form Pythagorean triplets.

For example: Take, $m = 2$, $n = 1$.

Then, $a = m^2 - n^2 = 3$, $b = 2mn = 4$, $c = m^2 + n^2 = 5$, is a Pythagorean triplet. (Check it!)

For, $m = 3$, $n = 2$, we get,

$a = 5$, $b = 12$, $c = 13$ which is again a Pythagorean triplet.

Take some more values for m and n and generate more such triplets.

2. When water freezes its volume increases by 4%. What volume of water is required to make 221 cm^3 of ice?

3. If price of tea increased by 20%, by what per cent must the consumption be reduced to keep the expense the same?

4. Ceremony Awards began in 1958. There were 28 categories to win an award. In 1993, there were 81 categories.

(i) The awards given in 1958 is what per cent of the awards given in 1993?

(ii) The awards given in 1993 is what per cent of the awards given in 1958?

5. Out of a swarm of bees, one fifth settled on a blossom of *Kadamba*, one third on a flower of *Silindhiri,* and three times the difference between these two numbers flew to the bloom of *Kutaja*. Only ten bees were then left from the swarm. What was the number of bees in the swarm? (Note, *Kadamba, Silindhiri* and *Kutaja* are flowering trees. The problem is from the ancient Indian text on algebra.)

6. In computing the area of a square, Shekhar used the formula for area of a square, while his friend Maroof used the formula for the perimeter of a square. Interestingly their answers were numerically same. Tell me the number of units of the side of the square they worked on.

7. The area of a square is numerically less than six times its side. List some squares in which this happens.

8. Is it possible to have a right circular cylinder to have volume numerically equal to its curved surface area? If yes state when.

9. Leela invited some friends for tea on her birthday. Her mother placed some plates and some *puris* on a table to be served. If Leela places 4 *puris* in each plate 1 plate would be left empty. But if she places 3 *puris* in each plate 1 *puri* would be left. Find the number of plates and number of *puris* on the table.

10. Is there a number which is equal to its cube but not equal to its square? If yes find it.

11. Arrange the numbers from 1 to 20 in a row such that the sum of any two adjacent numbers is a perfect square.

© NCERT not to be republished

Answers

2. $212\frac{1}{2}\ \text{cm}^3$

3. $16\frac{2}{3}\%$

4. (i) 34.5% (ii) 289%

5. 150

6. 4 units

7. Sides = 1, 2, 3, 4, 5 units

8. Yes, when radius = 2 units

9. Number of *puris* = 16, number of plates = 5

10. – 1

11. One of the ways is, 1, 3, 6, 19, 17, 8 (1 + 3 = 4, 3 + 6 = 9 etc.). Try some other ways.

© NCERT not to be republished

NOTES

© NCERT not to be republished

CHAPTER

1

Rational Numbers

0852CH01

1.1 Introduction

In Mathematics, we frequently come across simple equations to be solved. For example, the equation

$$x + 2 = 13 \quad (1)$$

is solved when $x = 11$, because this value of x satisfies the given equation. The solution 11 is a **natural number**. On the other hand, for the equation

$$x + 5 = 5 \quad (2)$$

the solution gives the **whole number** 0 (zero). If we consider only natural numbers, equation (2) cannot be solved. To solve equations like (2), we added the number zero to the collection of natural numbers and obtained the whole numbers. Even whole numbers will not be sufficient to solve equations of type

$$x + 18 = 5 \quad (3)$$

Do you see 'why'? We require the number –13 which is not a whole number. This led us to think of **integers, (positive and negative)**. Note that the positive integers correspond to natural numbers. One may think that we have enough numbers to solve all simple equations with the available list of integers. Now consider the equations

$$2x = 3 \quad (4)$$

$$5x + 7 = 0 \quad (5)$$

for which we cannot find a solution from the integers. (Check this)

We need the numbers $\frac{3}{2}$ to solve equation (4) and $\frac{-7}{5}$ to solve equation (5). This leads us to the collection of **rational numbers**.

We have already seen basic operations on rational numbers. We now try to explore some properties of operations on the different types of numbers seen so far.

© NCERT not to be republished

1.2 Properties of Rational Numbers

1.2.1 Closure

(i) Whole numbers

Let us revisit the closure property for all the operations on whole numbers in brief.

Operation	Numbers	Remarks
Addition	$0 + 5 = 5$, a whole number $4 + 7 = \ldots$. Is it a whole number? In general, $a + b$ is a whole number for any two whole numbers a and b.	Whole numbers are closed under addition.
Subtraction	$5 - 7 = -2$, which is not a whole number.	Whole numbers are **not** closed under subtraction.
Multiplication	$0 \times 3 = 0$, a whole number $3 \times 7 = \ldots$. Is it a whole number? In general, if a and b are any two whole numbers, their product ab is a whole number.	Whole numbers are closed under multiplication.
Division	$5 \div 8 = \frac{5}{8}$, which is not a whole number.	Whole numbers are **not** closed under division.

Check for closure property under all the four operations for natural numbers.

(ii) Integers

Let us now recall the operations under which integers are closed.

Operation	Numbers	Remarks
Addition	$-6 + 5 = -1$, an integer Is $-7 + (-5)$ an integer? Is $8 + 5$ an integer? In general, $a + b$ is an integer for any two integers a and b.	Integers are closed under addition.
Subtraction	$7 - 5 = 2$, an integer Is $5 - 7$ an integer? $-6 - 8 = -14$, an integer	Integers are closed under subtraction.

© NCERT not to be republished

	$-6-(-8)=2$, an integer Is $8-(-6)$ an integer? In general, for any two integers a and b, $a-b$ is again an integer. Check if $b-a$ is also an integer.	
Multiplication	$5\times 8=40$, an integer Is -5×8 an integer? $-5\times(-8)=40$, an integer In general, for any two integers a and b, $a\times b$ is also an integer.	Integers are closed under multiplication.
Division	$5\div 8=\frac{5}{8}$, which is not an integer.	Integers are **not** closed under division.

You have seen that whole numbers are closed under addition and multiplication but not under subtraction and division. However, integers are closed under addition, subtraction and multiplication but not under division.

(iii) Rational numbers

Recall that a number which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a **rational number**. For example, $-\frac{2}{3}, \frac{6}{7}, \frac{9}{-5}$ are all rational numbers. Since the numbers 0, –2, 4 can be written in the form $\frac{p}{q}$, they are also rational numbers. (Check it!)

(a) You know how to add two rational numbers. Let us add a few pairs.

$$\frac{3}{8}+\frac{(-5)}{7}=\frac{21+(-40)}{56}=\frac{-19}{56}$$ (a rational number)

$$\frac{-3}{8}+\frac{(-4)}{5}=\frac{-15+(-32)}{40}=\ldots$$ Is it a rational number?

$$\frac{4}{7}+\frac{6}{11}=\ldots$$ Is it a rational number?

We find that sum of two rational numbers is again a rational number. Check it for a few more pairs of rational numbers.

We say that *rational numbers are closed under addition. That is, for any two rational numbers a and b, a + b is also a rational number.*

(b) Will the difference of two rational numbers be again a rational number?

We have,

$$\frac{-5}{7}-\frac{2}{3}=\frac{-5\times 3-2\times 7}{21}=\frac{-29}{21}$$ (a rational number)

© NCERT not to be republished

$\frac{5}{8} - \frac{4}{5} = \frac{25-32}{40} = \ldots$ Is it a rational number?

$\frac{3}{7} - \left(\frac{-8}{5}\right) = \ldots$ Is it a rational number?

Try this for some more pairs of rational numbers. We find that *rational numbers are closed under subtraction. That is, for any two rational numbers a and b, a – b is also a rational number.*

(c) Let us now see the product of two rational numbers.

$\frac{-2}{3} \times \frac{4}{5} = \frac{-8}{15}; \frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$ (both the products are rational numbers)

$-\frac{4}{5} \times \frac{-6}{11} = \ldots$ Is it a rational number?

Take some more pairs of rational numbers and check that their product is again a rational number.

We say that *rational numbers are closed under multiplication. That is, for any two rational numbers a and b, a × b is also a rational number.*

(d) We note that $\frac{-5}{3} \div \frac{2}{5} = \frac{-25}{6}$ (a rational number)

$\frac{2}{7} \div \frac{5}{3} = \ldots$. Is it a rational number? $\frac{-3}{8} \div \frac{-2}{9} = \ldots$. Is it a rational number?

Can you say that rational numbers are closed under division?

We find that for any rational number a, $a \div 0$ is **not defined**.

So rational numbers are **not closed** under division.

However, if we exclude zero then the collection of, all other rational numbers is closed under division.

TRY THESE

Fill in the blanks in the following table.

Numbers	Closed under			
	addition	subtraction	multiplication	division
Rational numbers	Yes	Yes	...	No
Integers	...	Yes	...	No
Whole numbers	...	...	Yes	...
Natural numbers	...	No	...	...

© NCERT not to be republished

1.2.2 Commutativity

(i) Whole numbers

Recall the commutativity of different operations for whole numbers by filling the following table.

Operation	Numbers	Remarks
Addition	$0 + 7 = 7 + 0 = 7$ $2 + 3 = \ldots + \ldots = \ldots.$ For any two whole numbers a and b, $a + b = b + a$	Addition is commutative.
Subtraction	.....	Subtraction is not commutative.
Multiplication	.....	Multiplication is commutative.
Division	.....	Division is not commutative.

Check whether the commutativity of the operations hold for natural numbers also.

(ii) Integers

Fill in the following table and check the commutativity of different operations for integers:

Operation	Numbers	Remarks
Addition	.....	Addition is commutative.
Subtraction	Is $5 - (-3) = -3 - 5$?	Subtraction is not commutative.
Multiplication	.....	Multiplication is commutative.
Division	.....	Division is not commutative.

(iii) Rational numbers

(a) Addition

You know how to add two rational numbers. Let us add a few pairs here.

$$\frac{-2}{3}+\frac{5}{7}=\frac{1}{21} \text{ and } \frac{5}{7}+\left(\frac{-2}{3}\right)=\frac{1}{21}$$

So, $$\frac{-2}{3}+\frac{5}{7}=\frac{5}{7}+\left(\frac{-2}{3}\right)$$

Also, $$\frac{-6}{5}+\left(\frac{-8}{3}\right) = \ldots \text{ and } \frac{-8}{3}+\left(\frac{-6}{5}\right)=\ldots$$

Is $$\frac{-6}{5}+\left(\frac{-8}{3}\right)=\left(\frac{-8}{3}\right)+\left(\frac{-6}{5}\right)?$$

© NCERT not to be republished

Is $\frac{-3}{8}+\frac{1}{7}=\frac{1}{7}+\left(\frac{-3}{8}\right)$?

You find that two *rational numbers can be added in any order. We say that addition is commutative for rational numbers. That is, for any two rational numbers a and b, a + b = b + a.*

(b) Subtraction

Is $\frac{2}{3}-\frac{5}{4}=\frac{5}{4}-\frac{2}{3}$?

Is $\frac{1}{2}-\frac{3}{5}=\frac{3}{5}-\frac{1}{2}$?

You will find that subtraction is not commutative for rational numbers.

Note that subtraction is not commutative for integers and integers are also rational numbers. So, subtraction will not be commutative for rational numbers too.

(c) Multiplication

We have, $\frac{-7}{3}\times\frac{6}{5}=\frac{-42}{15}=\frac{6}{5}\times\left(\frac{-7}{3}\right)$

Is $\frac{-8}{9}\times\left(\frac{-4}{7}\right)=\frac{-4}{7}\times\left(\frac{-8}{9}\right)$?

Check for some more such products.

You will find that *multiplication is commutative for rational numbers.*

In general, a × b = b × a for any two rational numbers a and b.

(d) Division

Is $\frac{-5}{4}\div\frac{3}{7}=\frac{3}{7}\div\left(\frac{-5}{4}\right)$?

You will find that expressions on both sides are not equal.

So division is **not commutative** for rational numbers.

TRY THESE

Complete the following table:

Numbers	Commutative for			
	addition	subtraction	multiplication	division
Rational numbers	Yes	...	...	...
Integers	...	No	...	...
Whole numbers	...	...	Yes	...
Natural numbers	...	...	...	No

© NCERT not to be republished

1.2.3 Associativity

(i) Whole numbers

Recall the associativity of the four operations for whole numbers through this table:

Operation	Numbers	Remarks
Addition	.....	Addition is associative
Subtraction	.....	Subtraction is **not** associative
Multiplication	Is $7 \times (2 \times 5) = (7 \times 2) \times 5$? Is $4 \times (6 \times 0) = (4 \times 6) \times 0$? For any three whole numbers a, b and c $a \times (b \times c) = (a \times b) \times c$	Multiplication is associative
Division	.....	Division is **not** associative

Fill in this table and verify the remarks given in the last column.
Check for yourself the associativity of different operations for natural numbers.

(ii) Integers

Associativity of the four operations for integers can be seen from this table

Operation	Numbers	Remarks
Addition	Is $(-2) + [3 + (-4)]$ $= [(-2) + 3)] + (-4)$? Is $(-6) + [(-4) + (-5)]$ $= [(-6) + (-4)] + (-5)$? For any three integers a, b and c $a + (b + c) = (a + b) + c$	Addition is associative
Subtraction	Is $5 - (7 - 3) = (5 - 7) - 3$?	Subtraction is **not** associative
Multiplication	Is $5 \times [(-7) \times (-8)$ $= [5 \times (-7)] \times (-8)$? Is $(-4) \times [(-8) \times (-5)]$ $= [(-4) \times (-8)] \times (-5)$? For any three integers a, b and c $a \times (b \times c) = (a \times b) \times c$	Multiplication is associative
Division	Is $[(-10) \div 2] \div (-5)$ $= (-10) \div [2 \div (-5)]$?	Division is **not** associative

© NCERT not to be republished

(iii) Rational numbers

(a) Addition

We have $\frac{-2}{3}+\left[\frac{3}{5}+\left(\frac{-5}{6}\right)\right]=\frac{-2}{3}+\left(\frac{-7}{30}\right)=\frac{-27}{30}=\frac{-9}{10}$

$$\left[\frac{-2}{3}+\frac{3}{5}\right]+\left(\frac{-5}{6}\right)=\frac{-1}{15}+\left(\frac{-5}{6}\right)=\frac{-27}{30}=\frac{-9}{10}$$

So, $\frac{-2}{3}+\left[\frac{3}{5}+\left(\frac{-5}{6}\right)\right]=\left[\frac{-2}{3}+\frac{3}{5}\right]+\left(\frac{-5}{6}\right)$

Find $\frac{-1}{2}+\left[\frac{3}{7}+\left(\frac{-4}{3}\right)\right]$ and $\left[\frac{-1}{2}+\frac{3}{7}\right]+\left(\frac{-4}{3}\right)$. Are the two sums equal?

Take some more rational numbers, add them as above and see if the two sums are equal. We find that *addition is associative for rational numbers. That is, for any three rational numbers* a, b and c, $a + (b + c) = (a + b) + c$.

(b) Subtraction

You already know that subtraction is not associative for integers, then what about rational numbers.

Is $\frac{-2}{3}-\left[\frac{-4}{5}-\frac{1}{2}\right]=\left[\frac{2}{3}-\left(\frac{-4}{5}\right)\right]-\frac{1}{2}$?

Check for yourself.

Subtraction is **not associative** for rational numbers.

(c) Multiplication

Let us check the associativity for multiplication.

$$\frac{-7}{3}\times\left(\frac{5}{4}\times\frac{2}{9}\right)=\frac{-7}{3}\times\frac{10}{36}=\frac{-70}{108}=\frac{-35}{54}$$

$$\left(\frac{-7}{3}\times\frac{5}{4}\right)\times\frac{2}{9}=\ldots$$

We find that $\frac{-7}{3}\times\left(\frac{5}{4}\times\frac{2}{9}\right)=\left(\frac{-7}{3}\times\frac{5}{4}\right)\times\frac{2}{9}$

Is $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{4}{5}\right)=\left(\frac{2}{3}\times\frac{-6}{7}\right)\times\frac{4}{5}$?

Take some more rational numbers and check for yourself.

We observe that *multiplication is associative for rational numbers. That is for any three rational numbers* a, b and c, $a \times (b \times c) = (a \times b) \times c$.

© NCERT not to be republished

(d) Division

Recall that division is not associative for integers, then what about rational numbers?

Let us see if $\frac{1}{2} \div \left[\frac{-1}{3} \div \frac{2}{5}\right] = \left[\frac{1}{2} \div \left(\frac{-1}{3}\right)\right] \div \frac{2}{5}$

We have, LHS $= \frac{1}{2} \div \left(\frac{-1}{3} \div \frac{2}{5}\right) = \frac{1}{2} \div \left(\frac{-1}{3} \times \frac{5}{2}\right)$ (reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$)

$= \frac{1}{2} \div \left(-\frac{5}{6}\right) = \ldots$

RHS $= \left[\frac{1}{2} \div \left(\frac{-1}{3}\right)\right] \div \frac{2}{5}$

$= \left(\frac{1}{2} \times \frac{-3}{1}\right) \div \frac{2}{5} = \frac{-3}{2} \div \frac{2}{5} = \ldots$

Is LHS = RHS? Check for yourself. You will find that division is **not associative** for rational numbers.

TRY THESE

Complete the following table:

Numbers	Associative for			
	addition	subtraction	multiplication	division
Rational numbers	...	...	...	No
Integers	...	...	Yes	...
Whole numbers	Yes	...	...	...
Natural numbers	...	No	...	...

Example 1: Find $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$

Solution: $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$

$= \frac{198}{462} + \left(\frac{-252}{462}\right) + \left(\frac{-176}{462}\right) + \left(\frac{105}{462}\right)$ (Note that 462 is the LCM of 7, 11, 21 and 22)

$= \frac{198 - 252 - 176 + 105}{462} = \frac{-125}{462}$

© NCERT not to be republished

We can also solve it as.

$$\frac{3}{7}+\left(\frac{-6}{11}\right)+\left(\frac{-8}{21}\right)+\frac{5}{22}$$

$$=\left[\frac{3}{7}+\left(\frac{-8}{21}\right)\right]+\left[\frac{-6}{11}+\frac{5}{22}\right]$$ (by using commutativity and associativity)

$$=\left[\frac{9+(-8)}{21}\right]+\left[\frac{-12+5}{22}\right]$$ (LCM of 7 and 21 is 21; LCM of 11 and 22 is 22)

$$=\frac{1}{21}+\left(\frac{-7}{22}\right)=\frac{22-147}{462}=\frac{-125}{462}$$

Do you think the properties of commutativity and associativity made the calculations easier?

Example 2: Find $\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$

Solution: We have

$$\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$$

$$=\left(-\frac{4\times3}{5\times7}\right)\times\left(\frac{15\times(-14)}{16\times9}\right)$$

$$=\frac{-12}{35}\times\left(\frac{-35}{24}\right)=\frac{-12\times(-35)}{35\times24}=\frac{1}{2}$$

We can also do it as.

$$\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$$

$$=\left(\frac{-4}{5}\times\frac{15}{16}\right)\times\left[\frac{3}{7}\times\left(\frac{-14}{9}\right)\right]$$ (Using commutativity and associativity)

$$=\frac{-3}{4}\times\left(\frac{-2}{3}\right)=\frac{1}{2}$$

1.2.4 The role of zero (0)

Look at the following.

$2 + 0 = 0 + 2 = 2$ (Addition of 0 to a whole number)

$-5 + 0 = \ldots + \ldots = -5$ (Addition of 0 to an integer)

$\frac{-2}{7} + \ldots = 0 + \left(\frac{-2}{7}\right) = \frac{-2}{7}$ (Addition of 0 to a rational number)

© NCERT not to be republished

You have done such additions earlier also. Do a few more such additions.

What do you observe? You will find that when you add 0 to a whole number, the sum is again that whole number. This happens for integers and rational numbers also.

In general, $a + 0 = 0 + a = a$, where a is a whole number

$b + 0 = 0 + b = b$, where b is an integer

$c + 0 = 0 + c = c$, where c is a rational number

Zero is called the identity for the addition of rational numbers. It is the additive identity for integers and whole numbers as well.

1.2.5 The role of 1

We have,

$5 \times 1 = 5 = 1 \times 5$ (Multiplication of 1 with a whole number)

$$\frac{-2}{7} \times 1 = \ldots \times \ldots = \frac{-2}{7}$$

$$\frac{3}{8} \times \ldots = 1 \times \frac{3}{8} = \frac{3}{8}$$

What do you find?

You will find that when you multiply any rational number with 1, you get back the same rational number as the product. Check this for a few more rational numbers. You will find that, $a \times 1 = 1 \times a = a$ for any rational number a.

We say *that* 1 *is the multiplicative identity for rational numbers.*

Is 1 the multiplicative identity for integers? For whole numbers?

THINK, DISCUSS AND WRITE

If a property holds for rational numbers, will it also hold for integers? For whole numbers? Which will? Which will not?

1.2.6 Distributivity of multiplication over addition for rational numbers

To understand this, consider the rational numbers $\frac{-3}{4}, \frac{2}{3}$ and $\frac{-5}{6}$.

$$\frac{-3}{4} \times \left\{\frac{2}{3} + \left(\frac{-5}{6}\right)\right\} = \frac{-3}{4} \times \left\{\frac{(4) + (-5)}{6}\right\}$$

$$= \frac{-3}{4} \times \left(\frac{-1}{6}\right) = \frac{3}{24} = \frac{1}{8}$$

Also $$\frac{-3}{4} \times \frac{2}{3} = \frac{-3 \times 2}{4 \times 3} = \frac{-6}{12} = \frac{-1}{2}$$

© NCERT not to be republished

And $\frac{-3}{4}\times\frac{-5}{6}=\frac{5}{8}$

Therefore $\left(\frac{-3}{4}\times\frac{2}{3}\right)+\left(\frac{-3}{4}\times\frac{-5}{6}\right)=\frac{-1}{2}+\frac{5}{8}=\frac{1}{8}$

Thus, $\frac{-3}{4}\times\left\{\frac{2}{3}+\frac{-5}{6}\right\}=\left(\frac{-3}{4}\times\frac{2}{3}\right)+\left(\frac{-3}{4}\times\frac{-5}{6}\right)$

Distributivity of Multiplication over Addition and Subtraction.
For all rational numbers a, b and c,
$a\,(b+c)=ab+ac$
$a\,(b-c)=ab-ac$

TRY THESE

Find using distributivity. (i) $\left\{\frac{7}{5}\times\left(\frac{-3}{12}\right)\right\}+\left\{\frac{7}{5}\times\frac{5}{12}\right\}$ (ii) $\left\{\frac{9}{16}\times\frac{4}{12}\right\}+\left\{\frac{9}{16}\times\frac{-3}{9}\right\}$

Example 3: Find $\frac{2}{5}\times\frac{-3}{7}-\frac{1}{14}-\frac{3}{7}\times\frac{3}{5}$

Solution:
$$\frac{2}{5}\times\frac{-3}{7}-\frac{1}{14}-\frac{3}{7}\times\frac{3}{5}=\frac{2}{5}\times\frac{-3}{7}-\frac{3}{7}\times\frac{3}{5}-\frac{1}{14}\quad\text{(by commutativity)}$$
$$=\frac{2}{5}\times\frac{-3}{7}+\left(\frac{-3}{7}\right)\times\frac{3}{5}-\frac{1}{14}$$
$$=\frac{-3}{7}\left(\frac{2}{5}+\frac{3}{5}\right)-\frac{1}{14}\quad\text{(by distributivity)}$$
$$=\frac{-3}{7}\times1-\frac{1}{14}=\frac{-6-1}{14}=\frac{-1}{2}$$

EXERCISE 1.1

1. Name the property under multiplication used in each of the following.

(i) $\frac{-4}{5}\times1=1\times\frac{-4}{5}=-\frac{4}{5}$ (ii) $-\frac{13}{17}\times\frac{-2}{7}=\frac{-2}{7}\times\frac{-13}{17}$

(iii) $\frac{-19}{29}\times\frac{29}{-19}=1$

2. Tell what property allows you to compute $\frac{1}{3}\times\left(6\times\frac{4}{3}\right)$ as $\left(\frac{1}{3}\times6\right)\times\frac{4}{3}$.

3. The product of two rational numbers is always a _____.

© NCERT not to be republished

WHAT HAVE WE DISCUSSED?

1. Rational numbers are **closed** under the operations of addition, subtraction and multiplication.
2. The operations addition and multiplication are
   (i) **commutative** for rational numbers.
   (ii) **associative** for rational numbers.
3. The rational number 0 is the **additive identity** for rational numbers.
4. The rational number 1 is the **multiplicative identity** for rational numbers.
5. **Distributivity** of rational numbers: For all rational numbers a, b and c,
   $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
6. Between any two given rational numbers there are countless rational numbers. The idea of **mean** helps us to find rational numbers between two rational numbers.

© NCERT not to be republished

NOTES

© NCERT not to be republished

CHAPTER

2

Linear Equations in One Variable

0852CH02

2.1 Introduction

In the earlier classes, you have come across several **algebraic expressions** and **equations**.

Some examples of expressions we have so far worked with are:

$$5x,\ 2x - 3,\ 3x + y,\ 2xy + 5,\ xyz + x + y + z,\ x^2 + 1,\ y + y^2$$

Some examples of equations are: $5x = 25,\ 2x - 3 = 9,\ 2y + \frac{5}{2} = \frac{37}{2},\ 6z + 10 = -2$

You would remember that equations use the *equality* (=) sign; it is missing in expressions.

Of these given expressions, many have more than one variable. For example, $2xy + 5$ has two variables. We however, restrict to expressions with only one variable when we form equations. Moreover, the expressions we use to form equations are linear. This means that the highest power of the variable appearing in the expression is 1.

These are linear expressions:

$$2x,\ 2x + 1,\ 3y - 7,\ 12 - 5z,\ \frac{5}{4}(x - 4) + 10$$

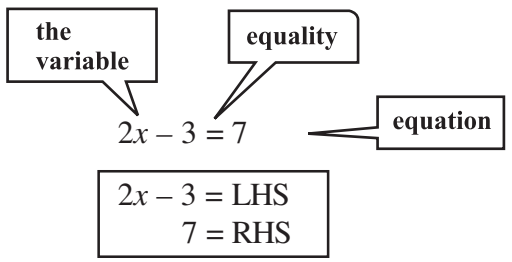
These are **not** linear expressions:

$x^2 + 1,\ y + y^2,\ 1 + z + z^2 + z^3$ (since highest power of variable > 1)

Here we will deal with equations with linear expressions in one variable only. Such equations are known as **linear equations in one variable**. The simple equations which you studied in the earlier classes were all of this type.

Let us briefly revise what we know:

(a) *An algebraic equation is an equality involving variables*. It has an *equality sign*. The expression on the left of the equality sign is the *Left Hand Side* (LHS). The expression on the right of the equality sign is the *Right Hand Side* (RHS).



© NCERT not to be republished

(b) In an equation the *values of the expressions on the LHS and RHS are equal*. This happens to be *true* only for certain values of the variable. These values are the **solutions** of the equation.

$x = 5$ is the solution of the equation $2x - 3 = 7$. For $x = 5$,
LHS $= 2 \times 5 - 3 = 7 =$ RHS
On the other hand $x = 10$ is not a solution of the equation. For $x = 10$, LHS $= 2 \times 10 - 3 = 17$. This is not equal to the RHS

(c) *How to find the solution of an equation?*

We assume that the two sides of the equation are balanced. We perform the same mathematical operations on both sides of the equation, so that the balance is not disturbed. A few such steps give the solution.

2.2 Solving Equations having the Variable on both Sides

An equation is the equality of the values of two expressions. In the equation $2x - 3 = 7$, the two expressions are $2x - 3$ and 7. In most examples that we have come across so far, the RHS is just a number. But this need not always be so; both sides could have expressions with variables. For example, the equation $2x - 3 = x + 2$ has expressions with a variable on both sides; the expression on the LHS is $(2x - 3)$ and the expression on the RHS is $(x + 2)$.

- We now discuss how to solve such equations which have expressions with the variable on both sides.

Example 1: Solve $2x - 3 = x + 2$

Solution: We have

$$2x = x + 2 + 3$$

or $$2x = x + 5$$

or $$2x - x = x + 5 - x \quad \text{(subtracting } x \text{ from both sides)}$$

or $$x = 5 \quad \text{(solution)}$$

Here we subtracted from both sides of the equation, not a number (constant), but a term involving the variable. We can do this as variables are also numbers. Also, note that subtracting x from both sides amounts to transposing x to LHS.

Example 2: Solve $5x + \frac{7}{2} = \frac{3}{2}x - 14$

Solution: Multiply both sides of the equation by 2. We get

$$2 \times \left(5x + \frac{7}{2}\right) = 2 \times \left(\frac{3}{2}x - 14\right)$$

© NCERT not to be republished

$$(2 \times 5x) + \left(2 \times \frac{7}{2}\right) = \left(2 \times \frac{3}{2}x\right) - (2 \times 14)$$

or $10x + 7 = 3x - 28$

or $10x - 3x + 7 = -28$ (transposing $3x$ to LHS)

or $7x + 7 = -28$

or $7x = -28 - 7$

or $7x = -35$

or $x = \frac{-35}{7}$ or $x = -5$ (solution)

EXERCISE 2.1

Solve the following equations and check your results.

1. $3x = 2x + 18$ **2.** $5t - 3 = 3t - 5$ **3.** $5x + 9 = 5 + 3x$

4. $4z + 3 = 6 + 2z$ **5.** $2x - 1 = 14 - x$ **6.** $8x + 4 = 3(x - 1) + 7$

7. $x = \frac{4}{5}(x + 10)$ **8.** $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$ **9.** $2y + \frac{5}{3} = \frac{26}{3} - y$

10. $3m = 5m - \frac{8}{5}$

2.3 Reducing Equations to Simpler Form

Example 16: Solve $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$

Why 6? Because it is the smallest multiple (or LCM) of the given denominators.

Solution: Multiplying both sides of the equation by 6,

$$\frac{6(6x+1)}{3} + 6 \times 1 = \frac{6(x-3)}{6}$$

or $2(6x + 1) + 6 = x - 3$

or $12x + 2 + 6 = x - 3$ (opening the brackets)

or $12x + 8 = x - 3$

or $12x - x + 8 = -3$

or $11x + 8 = -3$

or $11x = -3 - 8$

or $11x = -11$

or $x = -1$ (required solution)

© NCERT not to be republished

Check: LHS $= \frac{6(-1)+1}{3}+1 = \frac{-6+1}{3}+1 = \frac{-5}{3}+\frac{3}{3} = \frac{-5+3}{3} = \frac{-2}{3}$

RHS $= \frac{(-1)-3}{6} = \frac{-4}{6} = \frac{-2}{3}$

LHS = RHS. (as required)

Example 17: Solve $5x - 2(2x - 7) = 2(3x - 1) + \frac{7}{2}$

Solution: Let us open the brackets,

LHS $= 5x - 4x + 14 = x + 14$

RHS $= 6x - 2 + \frac{7}{2} = 6x - \frac{4}{2} + \frac{7}{2} = 6x + \frac{3}{2}$

The equation is $x + 14 = 6x + \frac{3}{2}$

or $14 = 6x - x + \frac{3}{2}$

or $14 = 5x + \frac{3}{2}$

or $14 - \frac{3}{2} = 5x$ (transposing $\frac{3}{2}$)

or $\frac{28-3}{2} = 5x$

or $\frac{25}{2} = 5x$

or $x = \frac{25}{2} \times \frac{1}{5} = \frac{5 \times 5}{2 \times 5} = \frac{5}{2}$

Did you observe how we simplified the form of the given equation? Here, we had to multiply both sides of the equation by the LCM of the denominators of the terms in the expressions of the equation.

Therefore, required solution is $x = \frac{5}{2}$.

***Check*:** LHS $= 5 \times \frac{5}{2} - 2\left(\frac{5}{2} \times 2 - 7\right)$

$= \frac{25}{2} - 2(5-7) = \frac{25}{2} - 2(-2) = \frac{25}{2} + 4 = \frac{25+8}{2} = \frac{33}{2}$

RHS $= 2\left(\frac{5}{2} \times 3 - 1\right) + \frac{7}{2} = 2\left(\frac{15}{2} - \frac{2}{2}\right) + \frac{7}{2} = \frac{2 \times 13}{2} + \frac{7}{2}$

$= \frac{26+7}{2} = \frac{33}{2} = \text{LHS}$. (as required)

Note, in this example we brought the equation to a simpler form by opening brackets and combining like terms on both sides of the equation.

© NCERT not to be republished

EXERCISE 2.2

Solve the following linear equations.

1. $\frac{x}{2}-\frac{1}{5}=\frac{x}{3}+\frac{1}{4}$ **2.** $\frac{n}{2}-\frac{3n}{4}+\frac{5n}{6}=21$ **3.** $x+7-\frac{8x}{3}=\frac{17}{6}-\frac{5x}{2}$

4. $\frac{x-5}{3}=\frac{x-3}{5}$ **5.** $\frac{3t-2}{4}-\frac{2t+3}{3}=\frac{2}{3}-t$ **6.** $m-\frac{m-1}{2}=1-\frac{m-2}{3}$

Simplify and solve the following linear equations.

7. $3(t-3)=5(2t+1)$ **8.** $15(y-4)-2(y-9)+5(y+6)=0$

9. $3(5z-7)-2(9z-11)=4(8z-13)-17$

10. $0.25(4f-3)=0.05(10f-9)$

WHAT HAVE WE DISCUSSED?

1. An algebraic equation is an equality involving variables. It says that the value of the expression on one side of the equality sign is equal to the value of the expression on the other side.
2. The equations we study in Classes VI, VII and VIII are linear equations in one variable. In such equations, the expressions which form the equation contain only one variable. Further, the equations are linear, i.e., the highest power of the variable appearing in the equation is 1.
3. An equation may have linear expressions on both sides. Equations that we studied in Classes VI and VII had just a number on one side of the equation.
4. Just as numbers, variables can, also, be transposed from one side of the equation to the other.
5. Occasionally, the expressions forming equations have to be simplified before we can solve them by usual methods. Some equations may not even be linear to begin with, but they can be brought to a linear form by multiplying both sides of the equation by a suitable expression.
6. The utility of linear equations is in their diverse applications; different problems on numbers, ages, perimeters, combination of currency notes, and so on can be solved using linear equations.

© NCERT not to be republished

NOTES

© NCERT not to be republished

CHAPTER

3

Understanding Quadrilaterals

0852CH03

3.1 Introduction

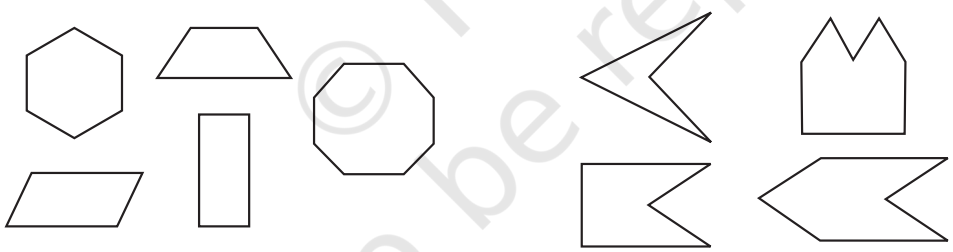
You know that the paper is a model for a **plane surface**. When you join a number of points without lifting a pencil from the paper (and without retracing any portion of the drawing other than single points), you get a **plane curve**.

3.1.1 Convex and concave polygons

A simple closed curve made up of only line segments is called a **polygon**.

Curves that are polygons **Curves that are not polygons**

Here are some convex polygons and some concave polygons. (Fig 3.1)



Convex polygons **Concave polygons**

Fig 3.1

Can you find how these types of polygons differ from one another? Polygons that are convex have no portions of their diagonals in their exteriors or any line segment joining any two different points, in the interior of the polygon, lies wholly in the interior of it . Is this true with concave polygons? Study the figures given. Then try to describe in your own words what we mean by a convex polygon and what we mean by a concave polygon. Give two rough sketches of each kind.

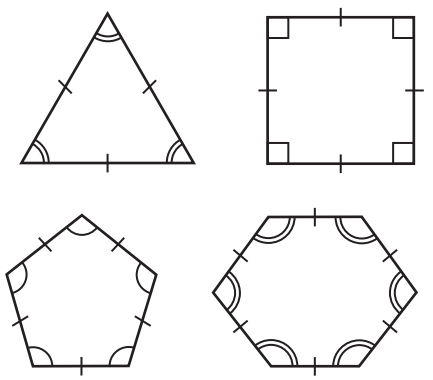
In our work in this class, we will be dealing with convex polygons only.

3.1.2 Regular and irregular polygons

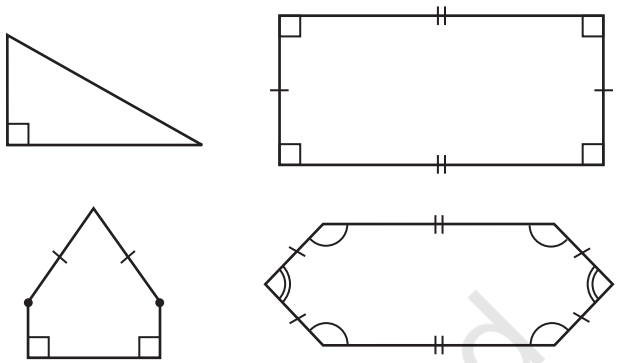
A regular polygon is both 'equiangular' and 'equilateral'. For example, a square has sides of equal length and angles of equal measure. Hence it is a regular polygon. A rectangle is

© NCERT not to be republished

equiangular but not equilateral. Is a rectangle a regular polygon? Is an equilateral triangle a regular polygon? Why?



Regular polygons



Polygons that are not regular

[**Note:** Use of or indicates segments of equal length].

In the previous classes, have you come across any quadrilateral that is equilateral but not equiangular? Recall the quadrilateral shapes you saw in earlier classes – Rectangle, Square, Rhombus etc.

Is there a triangle that is equilateral but not equiangular?

EXERCISE 3.1

1. Given here are some figures.

(1) (2) (3) (4)

(5) (6) (7) (8)

Classify each of them on the basis of the following.

(a) Simple curve (b) Simple closed curve (c) Polygon
(d) Convex polygon (e) Concave polygon

2. What is a regular polygon?

State the name of a regular polygon of

(i) 3 sides (ii) 4 sides (iii) 6 sides

© NCERT not to be republished

3.2 Sum of the Measures of the Exterior Angles of a Polygon

On many occasions a knowledge of exterior angles may throw light on the nature of interior angles and sides.

DO THIS

Draw a polygon on the floor, using a piece of chalk. (In the figure, a pentagon ABCDE is shown) (Fig 3.2).

We want to know the total measure of angles, i.e, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5$. Start at A. Walk along $\overline{AB}$. On reaching B, you need to turn through an angle of $m\angle 1$, to walk along. $\overline{BC}$ When you reach at C, you need to turn through an angle of $m\angle 2$ to walk along $\overline{CD}$. You continue to move in this manner, until you return to side AB. You would have in fact made one complete turn.

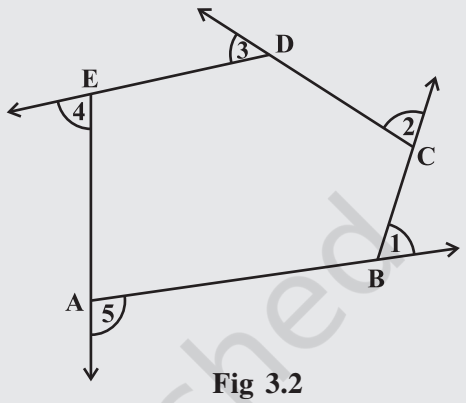


Fig 3.2

Therefore, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$.
This is true whatever be the number of sides of the polygon.

Therefore, *the sum of the measures of the external angles of any polygon is 360°.*

Example 1: Find measure x in Fig 3.3.

Solution: $x + 90° + 50° + 110° = 360°$ (Why?)

$x + 250° = 360°$

$x = 110°$

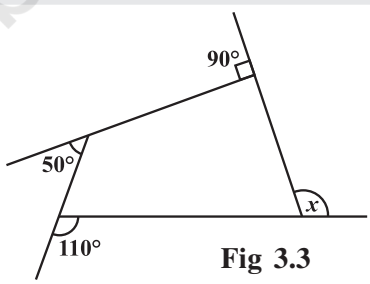


Fig 3.3

TRY THESE

Take a regular hexagon Fig 3.4.

1. What is the sum of the measures of its exterior angles x, y, z, p, q, r?
2. Is $x = y = z = p = q = r$? Why?
3. What is the measure of each?
   (i) exterior angle (ii) interior angle
4. Repeat this activity for the cases of
   (i) a regular octagon (ii) a regular 20-gon

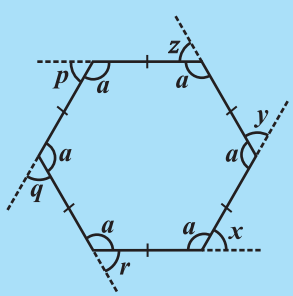


Fig 3.4

Example 2: Find the number of sides of a regular polygon whose each exterior angle has a measure of 45°.

Solution: Total measure of all exterior angles = 360°
Measure of each exterior angle = 45°

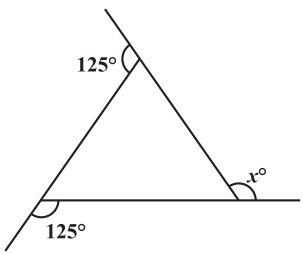
Therefore, the number of exterior angles = $\frac{360}{45} = 8$

The polygon has 8 sides.

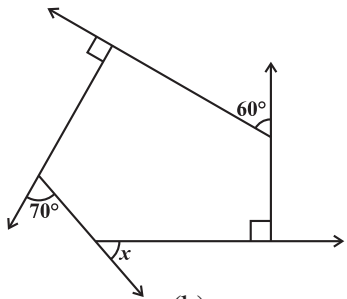
© NCERT not to be republished

EXERCISE 3.2

1. Find x in the following figures.



(a)



(b)

2. Find the measure of each exterior angle of a regular polygon of
(i) 9 sides (ii) 15 sides

3. How many sides does a regular polygon have if the measure of an exterior angle is 24°?

4. How many sides does a regular polygon have if each of its interior angles is 165°?

5. (a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?
(b) Can it be an interior angle of a regular polygon? Why?

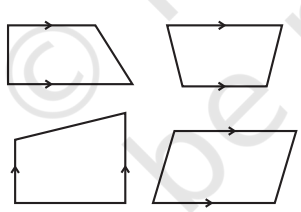
6. (a) What is the minimum interior angle possible for a regular polygon? Why?
(b) What is the maximum exterior angle possible for a regular polygon?

3.3 Kinds of Quadrilaterals

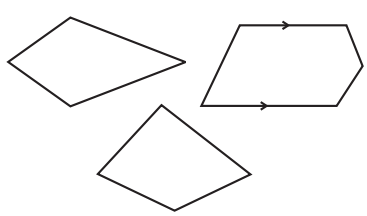
Based on the nature of the sides or angles of a quadrilateral, it gets special names.

3.3.1 Trapezium

Trapezium is a quadrilateral with a pair of parallel sides.



These are trapeziums



These are not trapeziums

Study the above figures and discuss with your friends why some of them are trapeziums while some are not. (**Note:** *The arrow marks indicate parallel lines*).

DO THIS

1. Take identical cut-outs of congruent triangles of sides 3 cm, 4 cm, 5 cm. Arrange them as shown (Fig 3.5).

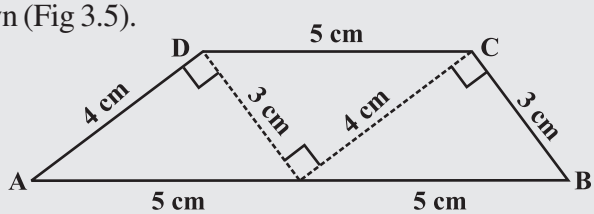


Fig 3.5

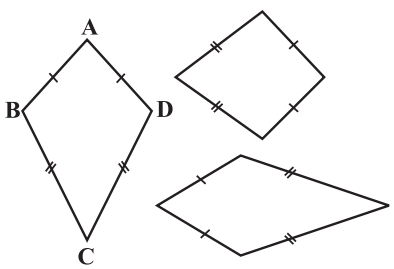
© NCERT not to be republished

You get a trapezium. (Check it!) Which are the parallel sides here? Should the non-parallel sides be equal?
You can get two more trapeziums using the same set of triangles. Find them out and discuss their shapes.

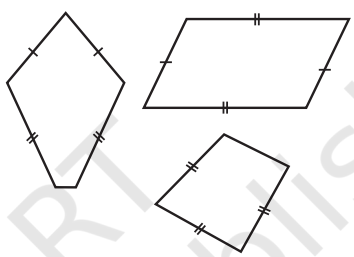
2. Take four set-squares from your and your friend's instrument boxes. Use different numbers of them to place side-by-side and obtain different trapeziums.
If the non-parallel sides of a trapezium are of equal length, we call it an *isosceles trapezium*. Did you get an isoceles trapezium in any of your investigations given above?

3.3.2 Kite

Kite is a special type of a quadrilateral. The sides with the same markings in each figure are equal. For example AB = AD and BC = CD.



These are kites



These are not kites

Study these figures and try to describe what a kite is. Observe that

(i) A kite has 4 sides (It is a quadrilateral).

(ii) There are exactly two **distinct consecutive pairs** of sides of equal length.

Check whether a square is a kite.

DO THIS

Take a thick white sheet.
Fold the paper once.
Draw two line segments of different lengths as shown in Fig 3.6.
Cut along the line segments and open up.
You have the shape of a kite (Fig 3.6).
Has the kite any line symmetry?

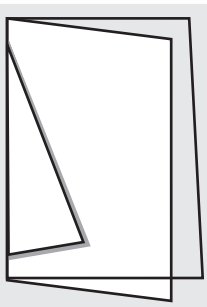


Fig 3.6

Show that ΔABC and ΔADC are congruent . What do we infer from this?

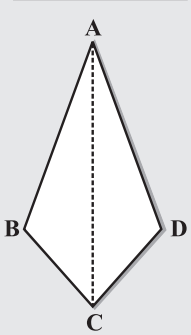


Fig 3.7

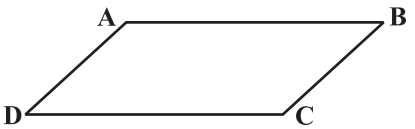
Fold both the diagonals of the kite. Use the set-square to check if they cut at right angles. Are the diagonals equal in length?
Verify (by paper-folding or measurement) if the diagonals bisect each other.
By folding an angle of the kite on its opposite, check for angles of equal measure.
Observe the diagonal folds; do they indicate any diagonal being an angle bisector?

Share your findings with others and list them. A summary of these results are given elsewhere in the chapter for your reference.

© NCERT not to be republished

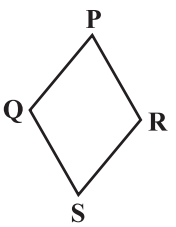
3.3.3 Parallelogram

A parallelogram is a quadrilateral. As the name suggests, it has something to do with parallel lines.



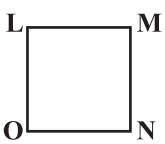
$\overline{AB} \parallel \overline{DC}$

$\overline{AD} \parallel \overline{BC}$



$\overline{QP} \parallel \overline{SR}$

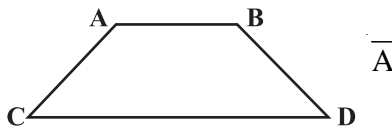
$\overline{QS} \parallel \overline{PR}$



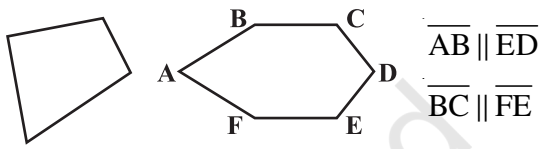
$\overline{LM} \parallel \overline{ON}$

$\overline{LO} \parallel \overline{MN}$

These are parallelograms



$\overline{AB} \parallel \overline{CD}$



$\overline{AB} \parallel \overline{ED}$

$\overline{BC} \parallel \overline{FE}$

These are not parallelograms

Study these figures and try to describe in your own words what we mean by a parallelogram. Share your observations with your friends.

Check whether a rectangle is also a parallelogram.

DO THIS

Take two different rectangular cardboard strips of different widths (Fig 3.8).

Strip 1 **Fig 3.8** **Strip 2**

Place one strip horizontally and draw lines along its edge as drawn in the figure (Fig 3.9).

Now place the other strip in a slant position over the lines drawn and use this to draw two more lines as shown (Fig 3.10).

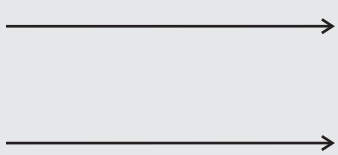


Fig 3.9

These four lines enclose a quadrilateral. This is made up of two pairs of parallel lines (Fig 3.11).

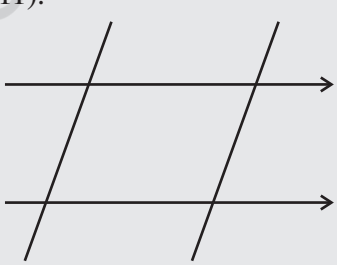


Fig 3.10

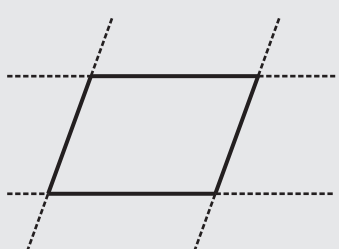


Fig 3.11

© NCERT not to be republished

It is a parallelogram.

A parallelogram is a quadrilateral whose opposite sides are parallel.

3.3.4 Elements of a parallelogram

There are four sides and four angles in a parallelogram. Some of these are equal. There are some terms associated with these elements that you need to remember.

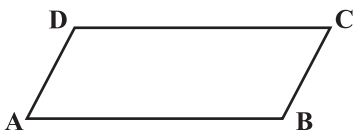


Fig 3.12

Given a parallelogram ABCD (Fig 3.12).

$\overline{AB}$ and $\overline{DC}$, are **opposite sides**. $\overline{AD}$ and $\overline{BC}$ form another pair of opposite sides.

$\angle A$ and $\angle C$ are a pair of **opposite angles**; another pair of opposite angles would be $\angle B$ and $\angle D$.

$\overline{AB}$ and $\overline{BC}$ are **adjacent sides**. This means, one of the sides starts where the other ends. Are $\overline{BC}$ and $\overline{CD}$ adjacent sides too? Try to find two more pairs of adjacent sides.

$\angle A$ and $\angle B$ are **adjacent angles**. They are at the ends of the same side. $\angle B$ and $\angle C$ are also adjacent. Identify other pairs of adjacent angles of the parallelogram.

DO THIS

Take cut-outs of two identical parallelograms, say ABCD and A′B′C′D′ (Fig 3.13).

Fig 3.13

Here $\overline{AB}$ is same as $\overline{A'B'}$ except for the name. Similarly the other corresponding sides are equal too.

Place $\overline{A'B'}$ over $\overline{DC}$. Do they coincide? What can you now say about the lengths $\overline{AB}$ and $\overline{DC}$?

Similarly examine the lengths $\overline{AD}$ and $\overline{BC}$. What do you find?

You may also arrive at this result by measuring $\overline{AB}$ and $\overline{DC}$.

Property: *The opposite sides of a parallelogram are of equal length.*

TRY THESE

Take two identical set squares with angles 30° – 60° – 90° and place them adjacently to form a parallelogram as shown in Fig 3.14. Does this help you to verify the above property?

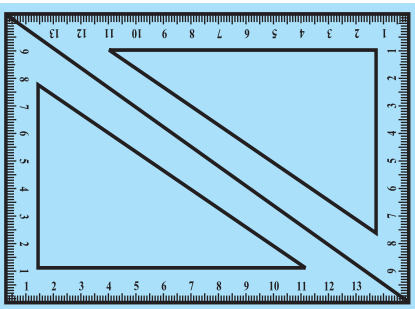


Fig 3.14

You can further strengthen this idea through a logical argument also.

Consider a parallelogram ABCD (Fig 3.15). Draw any one diagonal, say $\overline{AC}$.

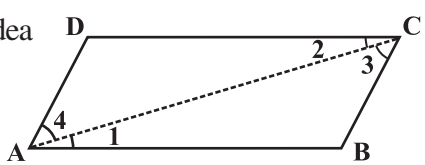


Fig 3.15

© NCERT not to be republished

Looking at the angles,

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ (Why?)

Since in triangles ABC and ADC, $\angle 1 = \angle 2$, $\angle 3 = \angle 4$

and $\overline{AC}$ is common, so, by ASA congruency condition,

$\Delta ABC \cong \Delta CDA$ (How is ASA used here?)

This gives $AB = DC$ and $BC = AD$.

Example 3: Find the perimeter of the parallelogram PQRS (Fig 3.16).

Solution: In a parallelogram, the opposite sides have same length.

Therefore, $PQ = SR = 12$ cm and $QR = PS = 7$ cm

So, Perimeter $= PQ + QR + RS + SP$

$= 12 \text{ cm} + 7 \text{ cm} + 12 \text{ cm} + 7 \text{ cm} = 38 \text{ cm}$

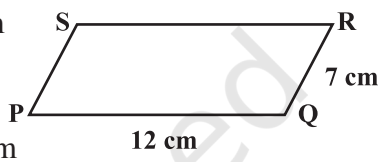


Fig 3.16

3.3.5 Angles of a parallelogram

We studied a property of parallelograms concerning the (opposite) sides. What can we say about the angles?

DO THIS

Let ABCD be a parallelogram (Fig 3.17). Copy it on a tracing sheet. Name this copy as A′B′C′D′. Place A′B′C′D′ on ABCD. Pin them together at the point where the diagonals meet. Rotate the transparent sheet by 180°. The parallelograms still concide; but you now find A′ lying exactly on C and vice-versa; similarly B′ lies on D and vice-versa.

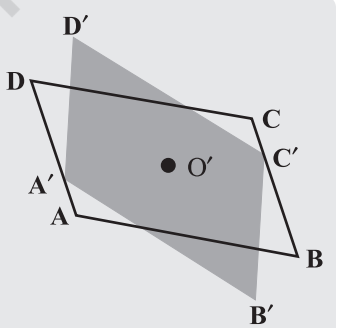


Fig 3.17

Does this tell you anything about the measures of the angles A and C? Examine the same for angles B and D. State your findings.

Property: *The opposite angles of a parallelogram are of equal measure.*

TRY THESE

Take two identical 30° – 60° – 90° set-squares and form a parallelogram as before. Does the figure obtained help you to confirm the above property?

You can further justify this idea through logical arguments.

If $\overline{AC}$ and $\overline{BD}$ are the diagonals of the parallelogram, (Fig 3.18) you find that

$\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ (Why?)

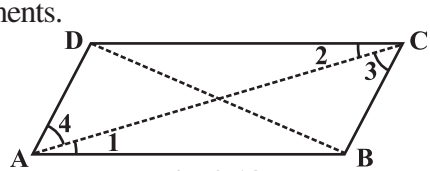


Fig 3.18

© NCERT not to be republished

Studying Δ ABC and Δ ADC (Fig 3.19) separately, will help you to see that by ASA congruency condition,

$$\Delta ABC \cong \Delta CDA \text{ (How?)}$$

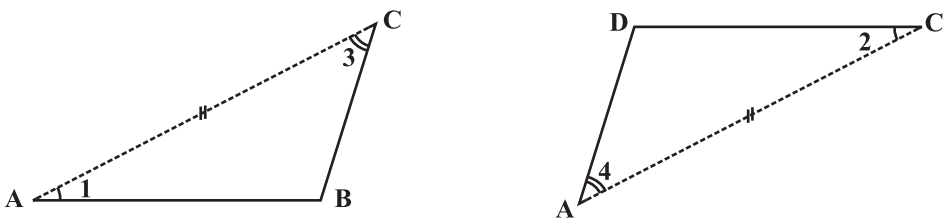


Fig 3.19

This shows that ∠B and ∠D have same measure. In the same way you can get $m\angle A = m\angle C$.

Alternatively, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, we have, $m\angle A = \angle 1 + \angle 4 = \angle 2 + \angle C$ $m\angle C$

Example 4: In Fig 3.20, BEST is a parallelogram. Find the values x, y and z.

Solution: S is opposite to B.

So, $x = 100°$ (opposite angles property)

$y = 100°$ (measure of angle corresponding to $\angle x$)

$z = 80°$ (since $\angle y$, $\angle z$ is a linear pair)

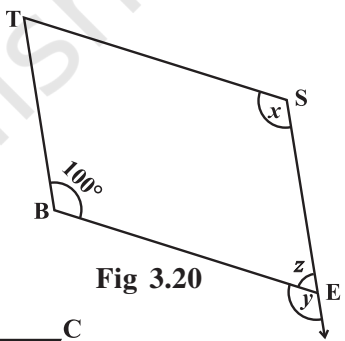


Fig 3.20

We now turn our attention to adjacent angles of a parallelogram.
In parallelogram ABCD, (Fig 3.21).

∠A and ∠D are supplementary since $\overline{DC} \parallel \overline{AB}$ and with transversal $\overline{DA}$, these two angles are interior opposite.

∠A and ∠B are also supplementary. Can you say 'why'?

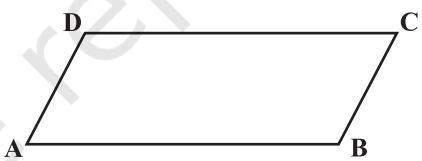


Fig 3.21

$\overline{AD} \parallel \overline{BC}$ and $\overline{BA}$ is a transversal, making ∠A and ∠B interior opposite.

Identify two more pairs of supplementary angles from the figure.

Property: *The adjacent angles in a parallelogram are supplementary.*

Example 5: In a parallelogram RING, (Fig 3.22) if $m\angle R = 70°$, find all the other angles.

Solution: Given $m\angle R = 70°$

Then $m\angle N = 70°$

because ∠R and ∠N are opposite angles of a parallelogram.

Since ∠R and ∠I are supplementary,

$$m\angle I = 180° - 70° = 110°$$

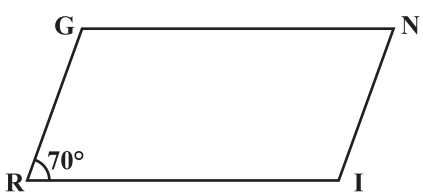


Fig 3.22

Also, $m\angle G = 110°$ since ∠G is opposite to ∠I

Thus, $m\angle R = m\angle N = 70°$ and $m\angle I = m\angle G = 110°$

© NCERT not to be republished

THINK, DISCUSS AND WRITE

After showing $m\angle R = m\angle N = 70°$, can you find $m\angle I$ and $m\angle G$ by any other method?

3.3.6 Diagonals of a parallelogram

The diagonals of a parallelogram, in general, are not of equal length. (Did you check this in your earlier activity?) However, the diagonals of a parallelogram have an interesting property.

DO THIS

Take a cut-out of a parallelogram, say, ABCD (Fig 3.23). Let its diagonals $\overline{AC}$ and $\overline{DB}$ meet at O.

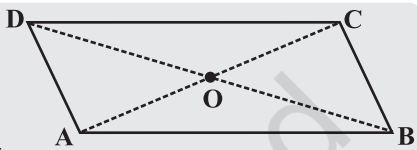


Fig 3.23

Find the mid point of $\overline{AC}$ by a fold, placing C on A. Is the mid-point same as O?

Does this show that diagonal $\overline{DB}$ bisects the diagonal $\overline{AC}$ at the point O? Discuss it with your friends. Repeat the activity to find where the mid point of $\overline{DB}$ could lie.

Property: *The diagonals of a parallelogram bisect each other (at the point of their intersection, of course!)*

To argue and justify this property is not very difficult. From Fig 3.24, applying ASA criterion, it is easy to see that

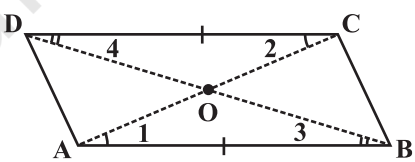


Fig 3.24

$\Delta AOB \cong \Delta COD$ (How is ASA used here?)

This gives $AO = CO$ and $BO = DO$

Example 6: In Fig 3.25 HELP is a parallelogram. (Lengths are in cms). Given that OE = 4 and HL is 5 more than PE? Find OH.

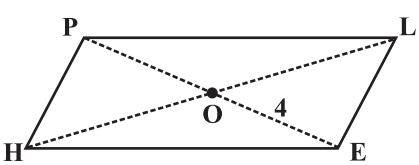


Fig 3.25

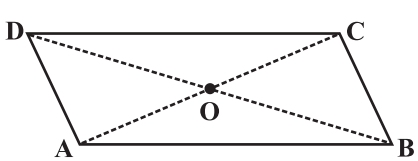
Solution : If OE = 4 then OP also is 4 (Why?)

So PE = 8, (Why?)

Therefore HL = 8 + 5 = 13

Hence $OH = \frac{1}{2} \times 13 = 6.5$ (cms)

EXERCISE 3.3



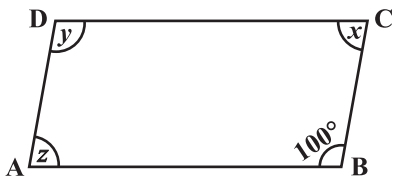
1. Given a parallelogram ABCD. Complete each statement along with the definition or property used.

   (i) AD = ... (ii) $\angle DCB$ = ...

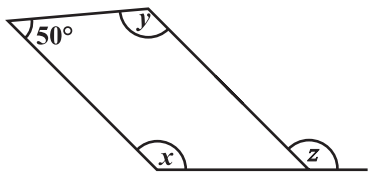
   (iii) OC = ... (iv) $m\angle DAB + m\angle CDA$ = ...

© NCERT not to be republished

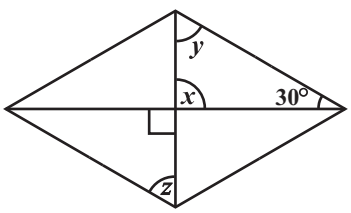
2. Consider the following parallelograms. Find the values of the unknowns x, y, z.



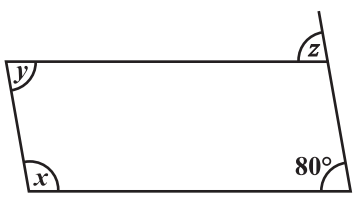
(i)



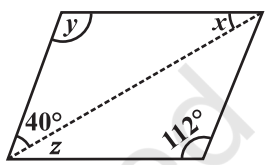
(ii)



(iii)



(iv)



(v)

3. Can a quadrilateral ABCD be a parallelogram if

(i) $\angle D + \angle B = 180°$? (ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?

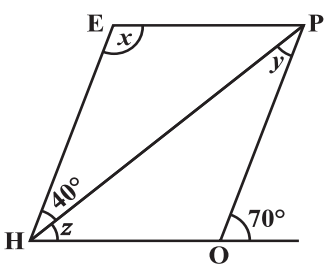
(iii) $\angle A = 70°$ and $\angle C = 65°$?

4. Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

5. The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the measure of each of the angles of the parallelogram.

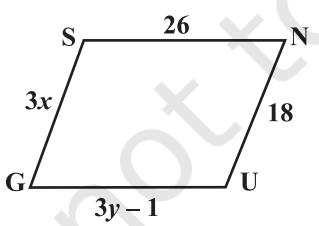
6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

7. The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.

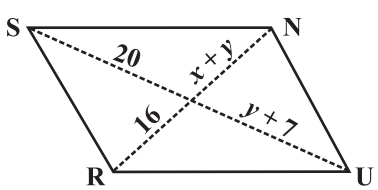


8. The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)

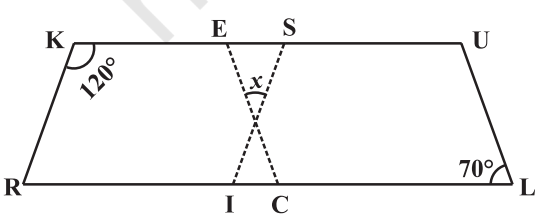
(i)



(ii)



9.



In the above figure both RISK and CLUE are parallelograms. Find the value of x.

© NCERT not to be republished

10. Explain how this figure is a trapezium. Which of its two sides are parallel? (Fig 3.26)

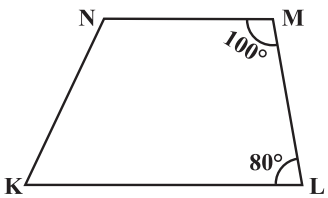


Fig 3.26

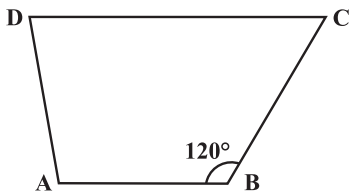


Fig 3.27

11. Find $m\angle C$ in Fig 3.27 if $\overline{AB} \parallel \overline{DC}$.

12. Find the measure of $\angle P$ and $\angle S$ if $\overline{SP} \parallel \overline{RQ}$ in Fig 3.28. (If you find $m\angle R$, is there more than one method to find $m\angle P$?)

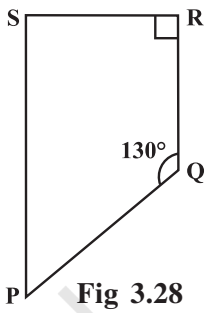


Fig 3.28

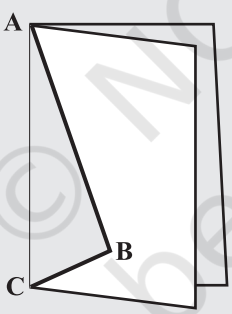
3.4 Some Special Parallelograms

3.4.1 Rhombus

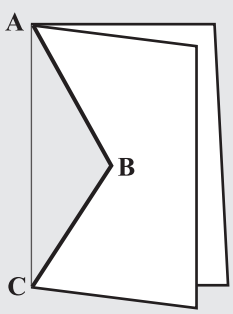
We obtain a Rhombus (which, you will see, is a parallelogram) as a special case of kite (which is not a a parallelogram).

DO THIS

Recall the paper-cut kite you made earlier.



Kite-cut



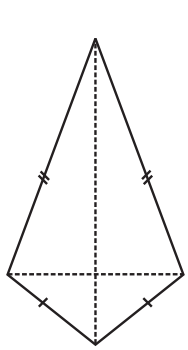
Rhombus-cut

When you cut along ABC and opened up, you got a kite. Here lengths AB and BC were different. If you draw AB = BC, then the kite you obtain is called a **rhombus**.

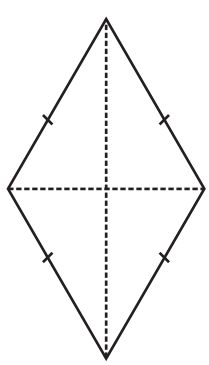
Note that the sides of rhombus are all of same length; this is not the case with the kite.

A rhombus is a quadrilateral with sides of equal length.

Since the opposite sides of a rhombus have the same length, it is also a parallelogram. So, *a rhombus has all the properties of a parallelogram and also that of a kite*. Try to list them out. You can then verify your list with the check list summarised in the book elsewhere.



Kite



Rhombus

© NCERT not to be republished

The most useful property of a rhombus is that of its diagonals.

Property: *The diagonals of a rhombus are perpendicular bisectors of one another.*

DO THIS

Take a copy of rhombus. By paper-folding verify if the point of intersection is the mid-point of each diagonal. You may also check if they intersect at right angles, using the corner of a set-square.

Here is an outline justifying this property using logical steps.

ABCD is a rhombus (Fig 3.29). Therefore it is a parallelogram too.

Since diagonals bisect each other, $OA = OC$ and $OB = OD$.

We have to show that $m\angle AOD = m\angle COD = 90°$

It can be seen that by SSS congruency criterion

$\Delta AOD \cong \Delta COD$

Therefore, $m\angle AOD = m\angle COD$

Since $\angle AOD$ and $\angle COD$ are a linear pair,

$m\angle AOD = m\angle COD = 90°$

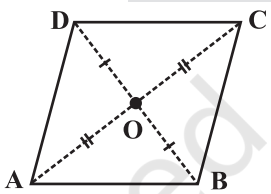


Fig 3.29

Since $AO = CO$ (Why?)

$AD = CD$ (Why?)

$OD = OD$

Example 7:

RICE is a rhombus (Fig 3.30). Find x, y, z. Justify your findings.

Solution:

$x = OE = OI$ (diagonals bisect) $= 5$

$y = OR = OC$ (diagonals bisect) $= 12$

$z =$ side of the rhombus $= 13$ (all sides are equal)

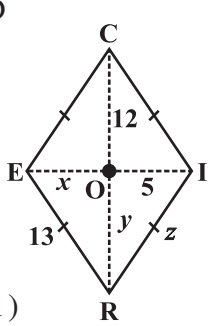


Fig 3.30

3.4.2 A rectangle

A rectangle is a parallelogram with equal angles (Fig 3.31).

What is the full meaning of this definition? Discuss with your friends.

If the rectangle is to be equiangular, what could be the measure of each angle?

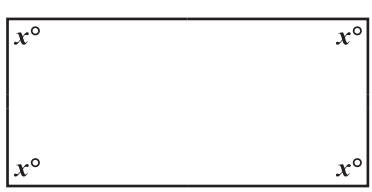


Fig 3.31

Let the measure of each angle be $x°$.

Then $4x° = 360°$ (Why)?

Therefore, $x° = 90°$

Thus each angle of a rectangle is a right angle.

So, a rectangle is a parallelogram in which every angle is a right angle.

Being a parallelogram, the rectangle has opposite sides of equal length and its diagonals bisect each other.

© NCERT not to be republished

In a parallelogram, the diagonals can be of different lengths. (Check this); but surprisingly the rectangle (being a special case) has diagonals of equal length.

Property: *The diagonals of a rectangle are of equal length.*

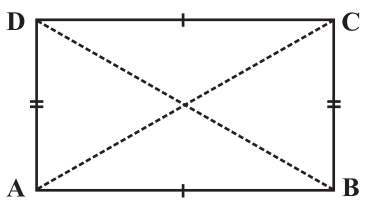


Fig 3.32

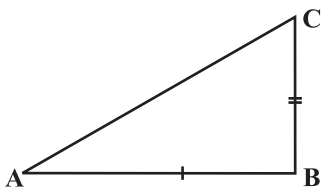


Fig 3.33

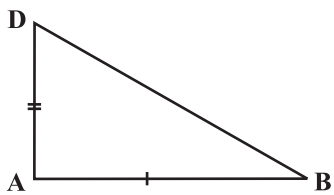


Fig 3.34

This is easy to justify. If ABCD is a rectangle (Fig 3.38), then looking at triangles ABC and ABD separately [(Fig 3.33) and (Fig 3.34) respectively], we have

$$\Delta ABC \cong \Delta ABD$$

This is because

$AB = AB$ (Common)

$BC = AD$ (Why?)

$m\angle A = m\angle B = 90°$ (Why?)

The congruency follows by SAS criterion.

Thus $AC = BD$

and *in a rectangle the diagonals, besides being equal in length bisect each other* (Why?)

Example 8: RENT is a rectangle (Fig 3.35). Its diagonals meet at O. Find x, if $OR = 2x + 4$ and $OT = 3x + 1$.

Solution: $\overline{OT}$ is half of the diagonal $\overline{TE}$,

$\overline{OR}$ is half of the diagonal $\overline{RN}$.

Diagonals are equal here. (Why?)

So, their halves are also equal.

Therefore $3x + 1 = 2x + 4$

or $x = 3$

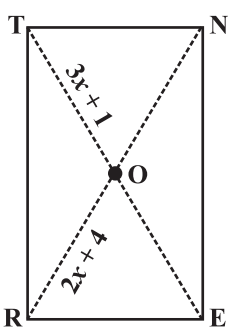


Fig 3.35

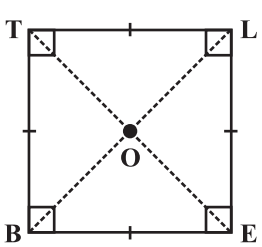
3.4.3 A square

A square is a rectangle with equal sides.

This means a square has all the properties of a rectangle with an additional requirement that all the sides have equal length.

The square, like the rectangle, has diagonals of equal length.

In a rectangle, there is no requirement for the diagonals to be perpendicular to one another, (Check this).



BELT is a square, $BE = EL = LT = TB$

$\angle B, \angle E, \angle L, \angle T$ are right angles.

$BL = ET$ and $\overline{BL} \perp \overline{ET}$.

$OB = OL$ and $OE = OT$.

© NCERT not to be republished

In a square the diagonals.

(i) bisect one another (square being a parallelogram)

(ii) are of equal length (square being a rectangle) and

(iii) are perpendicular to one another.

Hence, we get the following property.

Property: *The diagonals of a square are perpendicular bisectors of each other.*

DO THIS

Take a square sheet, say PQRS (Fig 3.37).

Fold along both the diagonals. Are their mid-points the same?

Check if the angle at O is 90° by using a set-square.

This verifies the property stated above.

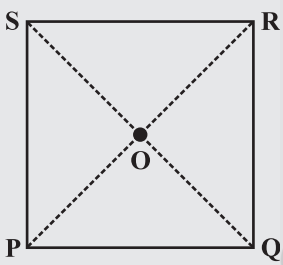


Fig 3.36

We can justify this also by arguing logically:

ABCD is a square whose diagonals meet at O (Fig 3.37).

$OA = OC$ (Since the square is a parallelogram)

By SSS congruency condition, we now see that

$\Delta AOD \cong \Delta COD$ (How?)

Therefore, $m\angle AOD = m\angle COD$

These angles being a linear pair, each is right angle.

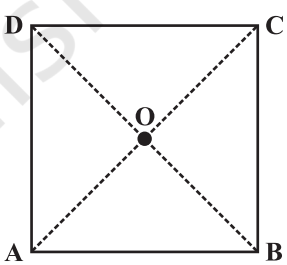


Fig 3.37

EXERCISE 3.4

1. State whether True or False.

(a) All rectangles are squares

(b) All rhombuses are parallelograms

(c) All squares are rhombuses and also rectangles

(d) All squares are not parallelograms.

(e) All kites are rhombuses.

(f) All rhombuses are kites.

(g) All parallelograms are trapeziums.

(h) All squares are trapeziums.

2. Identify all the quadrilaterals that have.

(a) four sides of equal length

(b) four right angles

3. Explain how a square is.

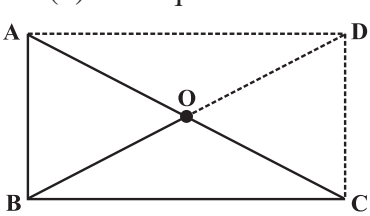
(i) a quadrilateral (ii) a parallelogram (iii) a rhombus (iv) a rectangle

4. Name the quadrilaterals whose diagonals.

(i) bisect each other (ii) are perpendicular bisectors of each other (iii) are equal

5. Explain why a rectangle is a convex quadrilateral.

6. ABC is a right-angled triangle and O is the mid point of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).

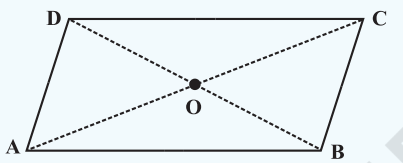
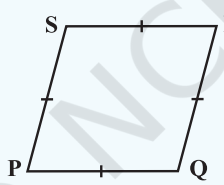
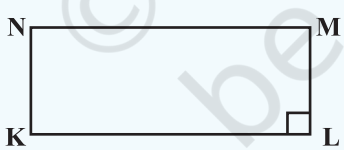
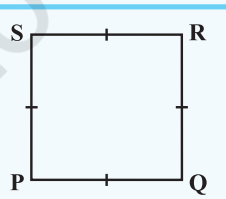
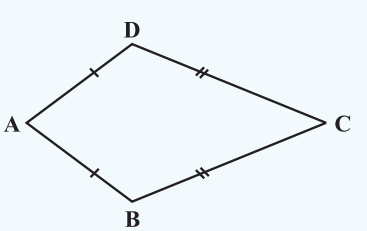


© NCERT not to be republished

THINK, DISCUSS AND WRITE

1. A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular?
2. A square was defined as a rectangle with all sides equal. Can we define it as rhombus with equal angles? Explore this idea.
3. Can a trapezium have all angles equal? Can it have all sides equal? Explain.

WHAT HAVE WE DISCUSSED?

Quadrilateral	Properties
Parallelogram: A quadrilateral with each pair of opposite sides parallel. 	(1) Opposite sides are equal. (2) Opposite angles are equal. (3) Diagonals bisect one another.
Rhombus: A parallelogram with sides of equal length. 	(1) All the properties of a parallelogram. (2) Diagonals are perpendicular to each other.
Rectangle: A parallelogram with a right angle. 	(1) All the properties of a parallelogram. (2) Each of the angles is a right angle. (3) Diagonals are equal.
Square: A rectangle with sides of equal length. 	All the properties of a parallelogram, rhombus and a rectangle.
Kite: A quadrilateral with exactly two pairs of equal consecutive sides 	(1) The diagonals are perpendicular to one another (2) One of the diagonals bisects the other. (3) In the figure $m\angle B = m\angle D$ but $m\angle A \neq m\angle C$.

© NCERT not to be republished

CHAPTER

4

Data Handling

0852CH05

4.1 Looking for Information

In your day-to-day life, you might have come across information, such as:

(a) Runs made by a batsman in the last 10 test matches.

(b) Number of wickets taken by a bowler in the last 10 ODIs.

(c) Marks scored by the students of your class in the Mathematics unit test.

(d) Number of story books read by each of your friends etc.

The information collected in all such cases is called **data**. Data is usually collected in the context of a situation that we want to study. For example, a teacher may like to know the average height of students in her class. To find this, she will write the heights of all the students in her class, organise the data in a systematic manner and then interpret it accordingly.

Sometimes, data is represented **graphically** to give a clear idea of what it represents. Do you remember the different types of graphs which we have learnt in earlier classes?

1. **A Pictograph:** Pictorial representation of data using symbols.

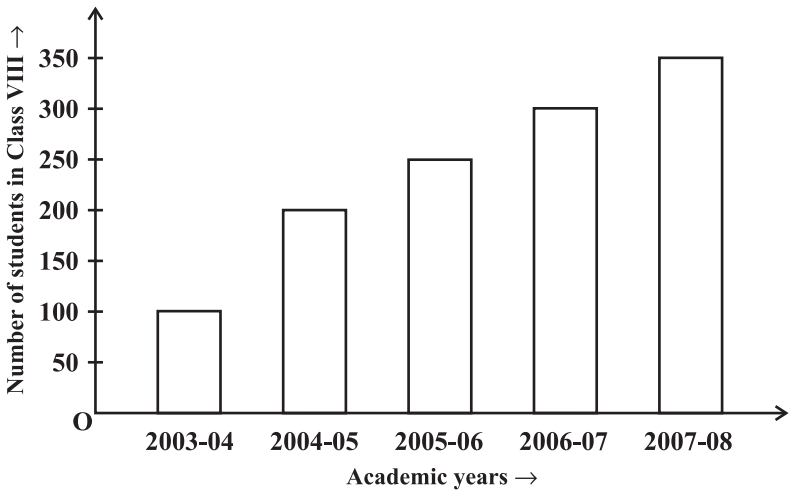
= 100 cars ← One symbol stands for 100 cars	
July	= 250 denotes $\frac{1}{2}$ of 100
August	= 300
September	= ?

(i) How many cars were produced in the month of July?

(ii) In which month were maximum number of cars produced?

2. **A bar graph:** A display of information using bars of uniform width, their heights being proportional to the respective values.

© NCERT not to be republished



Bar heights give the quantity for each category.

Bars are of equal width with equal gaps in between.

(i) What is the information given by the bar graph?

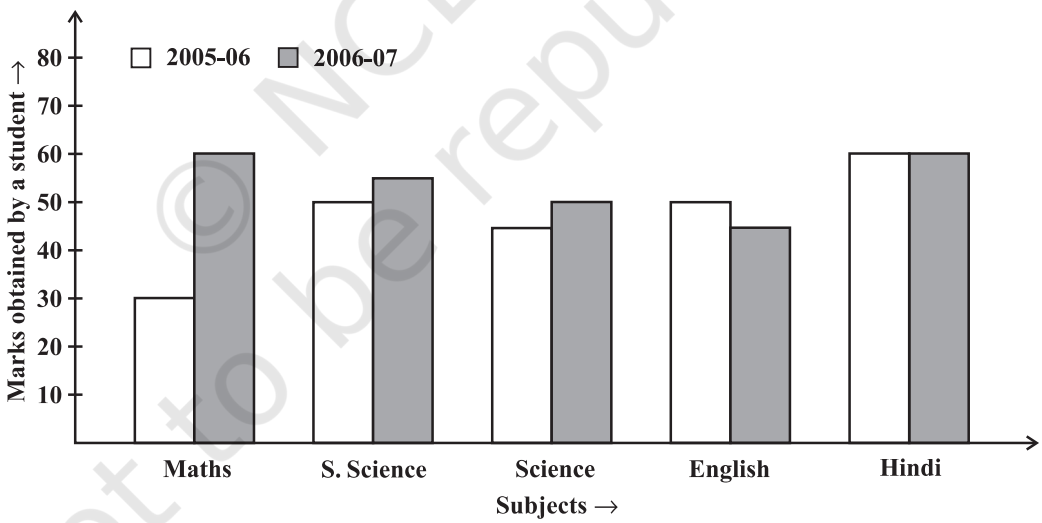
(ii) In which year is the increase in the number of students maximum?

(iii) In which year is the number of students maximum?

(iv) State whether true or false:

'The number of students during 2005-06 is twice that of 2003-04.'

3. **Double Bar Graph:** A bar graph showing two sets of data simultaneously. It is useful for the comparison of the data.



(i) What is the information given by the double bar graph?

(ii) In which subject has the performance improved the most?

(iii) In which subject has the performance deteriorated?

(iv) In which subject is the performance at par?

THINK, DISCUSS AND WRITE

If we change the position of any of the bars of a bar graph, would it change the information being conveyed? Why?

© NCERT not to be republished

TRY THESE

Draw an appropriate graph to represent the given information.

1.

Month	July	August	September	October	November	December
Number of watches sold	1000	1500	1500	2000	2500	1500

2.

Children who prefer	School A	School B	School C
Walking	40	55	15
Cycling	45	25	35

3. Percentage wins in ODI by 8 top cricket teams.

Teams	From Champions Trophy to World Cup-06	Last 10 ODI in 07
South Africa	75%	78%
Australia	61%	40%
Sri Lanka	54%	38%
New Zealand	47%	50%
England	46%	50%
Pakistan	45%	44%
West Indies	44%	30%
India	43%	56%

© NCERT not to be republished

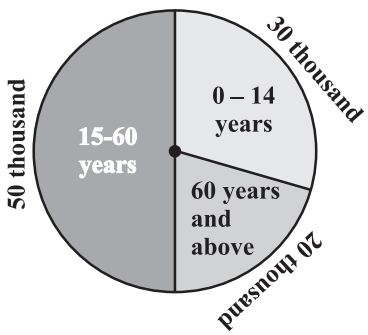
4.2 Circle Graph or Pie Chart

Have you ever come across data represented in circular form as shown (Fig 4.1)?

The time spent by a child during a day

(i)

Age groups of people in a town



(ii)

Fig 4.1

These are called **circle graphs**. A circle graph shows the relationship between a whole and its parts. Here, the whole circle is divided into sectors. The size of each sector is proportional to the activity or information it represents.

For example, in the above graph, the proportion of the sector for hours spent in sleeping

$$= \frac{\text{number of sleeping hours}}{\text{whole day}} = \frac{8 \text{ hours}}{24 \text{ hours}} = \frac{1}{3}$$

So, this sector is drawn as $\frac{1}{3}$rd part of the circle. Similarly, the proportion of the sector for hours spent in school $= \frac{\text{number of school hours}}{\text{whole day}} = \frac{6 \text{ hours}}{24 \text{ hours}} = \frac{1}{4}$

So this sector is drawn $\frac{1}{4}$th of the circle. Similarly, the size of other sectors can be found.

Add up the fractions for all the activities. Do you get the total as one?

A circle graph is also called a **pie chart**.

TRY THESE

1. Each of the following pie charts (Fig 4.2) gives you a different piece of information about your class. Find the fraction of the circle representing each of these information.

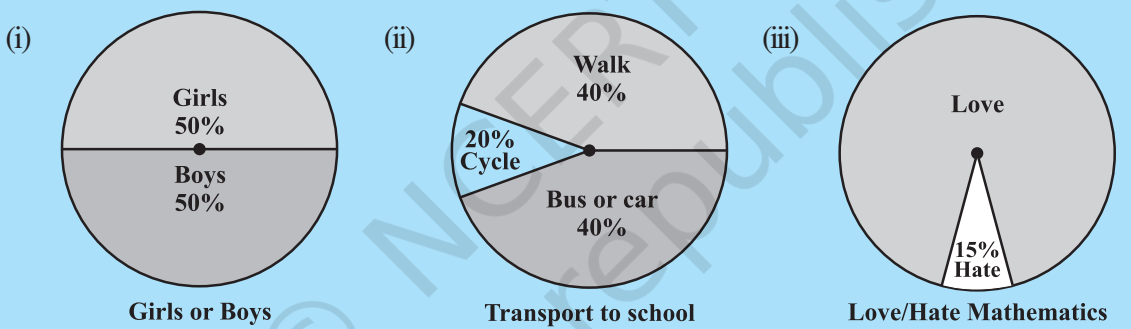
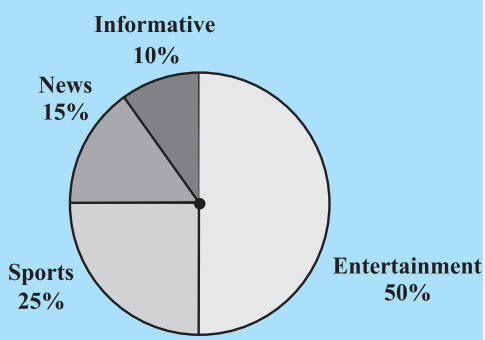


Fig 4.2

2. Answer the following questions based on the pie chart given (Fig 4.3).
   (i) Which type of programmes are viewed the most?
   (ii) Which two types of programmes have number of viewers equal to those watching sports channels?



Viewers watching different types of channels on T.V.

Fig 4.3

4.2.1 Drawing pie charts

The favourite flavours of ice-creams for students of a school is given in percentages as follows.

Flavours	Percentage of students Preferring the flavours
Chocolate	50%
Vanilla	25%
Other flavours	25%

© NCERT not to be republished

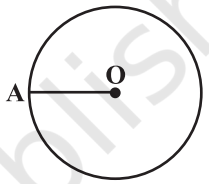
Let us represent this data in a pie chart.

The total angle at the centre of a circle is 360°. The central angle of the sectors will be a fraction of 360°. We make a table to find the central angle of the sectors (Table 4.1).

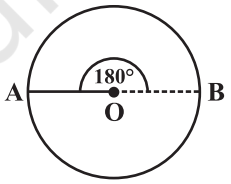
Table 4.1

Flavours	Students in per cent preferring the flavours	In fractions	Fraction of 360°
Chocolate	50%	$\frac{50}{100}=\frac{1}{2}$	$\frac{1}{2}$ of 360° = 180°
Vanilla	25%	$\frac{25}{100}=\frac{1}{4}$	$\frac{1}{4}$ of 360° = 90°
Other flavours	25%	$\frac{25}{100}=\frac{1}{4}$	$\frac{1}{4}$ of 360° = 90°

1. Draw a circle with any convenient radius.
   Mark its centre (O) and a radius (OA).



2. The angle of the sector for chocolate is 180°.
   Use the protractor to draw ∠AOB = 180°.



3. Continue marking the remaining sectors.

Example 1: Adjoining pie chart (Fig 4.4) gives the expenditure (in percentage) on various items and savings of a family during a month.

(i) On which item, the expenditure was maximum?
(ii) Expenditure on which item is equal to the total savings of the family?
(iii) If the monthly savings of the family is ₹3000, what is the monthly expenditure on clothes?

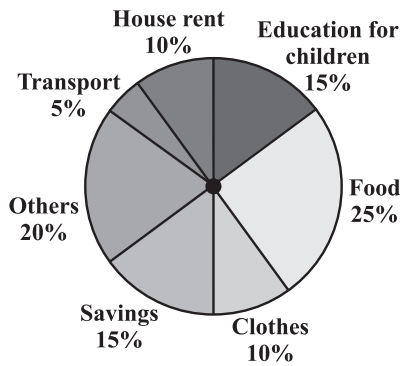


Fig 4.4

Solution:

(i) Expenditure is maximum on food.
(ii) Expenditure on Education of children is the same (i.e., 15%) as the savings of the family.

© NCERT not to be republished

(iii) 15% represents ₹ 3000

Therefore, 10% represents ₹ $\frac{3000}{15} \times 10 = ₹ 2000$

Example 2: On a particular day, the sales (in rupees) of different items of a baker's shop are given below.

ordinary bread	: 320
fruit bread	: 80
cakes and pastries	: 160
biscuits	: 120
others	: 40
Total	**: 720**

Draw a pie chart for this data.

Solution: We find the central angle of each sector. Here the total sale = ₹ 720. We thus have this table.

Item	Sales (in ₹)	In Fraction	Central Angle
Ordinary Bread	320	$\frac{320}{720} = \frac{4}{9}$	$\frac{4}{9} \times 360° = 160°$
Biscuits	120	$\frac{120}{720} = \frac{1}{6}$	$\frac{1}{6} \times 360° = 60°$
Cakes and pastries	160	$\frac{160}{720} = \frac{2}{9}$	$\frac{2}{9} \times 360° = 80°$
Fruit Bread	80	$\frac{80}{720} = \frac{1}{9}$	$\frac{1}{9} \times 360° = 40°$
Others	40	$\frac{40}{720} = \frac{1}{18}$	$\frac{1}{18} \times 360° = 20°$

Now, we make the pie chart (Fig 4.5):

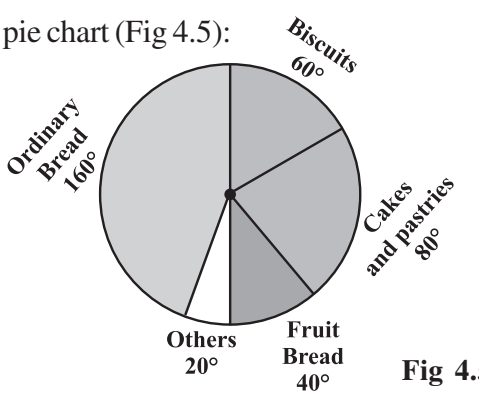


Fig 4.5

© NCERT not to be republished

TRY THESE

Draw a pie chart of the data given below.

The time spent by a child during a day.

Sleep — 8 hours
School — 6 hours
Home work — 4 hours
Play — 4 hours
Others — 2 hours

THINK, DISCUSS AND WRITE

Which form of graph would be appropriate to display the following data.

1. Production of food grains of a state.

Year	2001	2002	2003	2004	2005	2006
Production (in lakh tons)	60	50	70	55	80	85

2. Choice of food for a group of people.

Favourite food	**Number of people**
North Indian	30
South Indian	40
Chinese	25
Others	25
Total	**120**

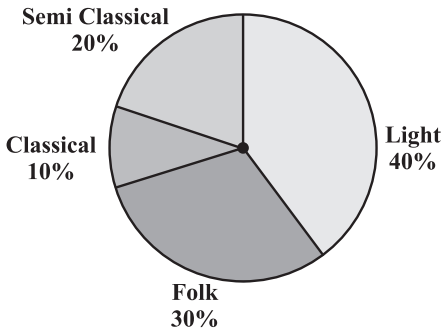
3. The daily income of a group of a factory workers.

Daily Income (in Rupees)	**Number of workers (in a factory)**
75-100	45
100-125	35
125-150	55
150-175	30
175-200	50
200-225	125
225-250	140
Total	**480**

© NCERT not to be republished

EXERCISE 4.1

1. A survey was made to find the type of music that a certain group of young people liked in a city. Adjoining pie chart shows the findings of this survey.



From this pie chart answer the following:

(i) If 20 people liked classical music, how many young people were surveyed?

(ii) Which type of music is liked by the maximum number of people?

(iii) If a cassette company were to make 1000 CD's, how many of each type would they make?

2. A group of 360 people were asked to vote for their favourite season from the three seasons rainy, winter and summer.

Season	No. of votes
Summer	90
Rainy	120
Winter	150

(i) Which season got the most votes?

(ii) Find the central angle of each sector.

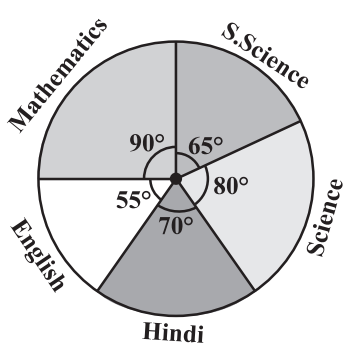
(iii) Draw a pie chart to show this information.

3. Draw a pie chart showing the following information. The table shows the colours preferred by a group of people.

Colours	Number of people
Blue	18
Green	9
Red	6
Yellow	3
Total	**36**

Find the proportion of each sector. For example, Blue is $\frac{18}{36} = \frac{1}{2}$; Green is $\frac{9}{36} = \frac{1}{4}$ and so on. Use this to find the corresponding angles.

4. The adjoining pie chart gives the marks scored in an examination by a student in Hindi, English, Mathematics, Social Science and Science. If the total marks obtained by the students were 540, answer the following questions.



(i) In which subject did the student score 105 marks?
(**Hint:** for 540 marks, the central angle = 360°. So, for 105 marks, what is the central angle?)

(ii) How many more marks were obtained by the student in Mathematics than in Hindi?

(iii) Examine whether the sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi.
(**Hint:** Just study the central angles).

© NCERT not to be republished

5. The number of students in a hostel, speaking different languages is given below. Display the data in a pie chart.

Language	Hindi	English	Marathi	Tamil	Bengali	Total
Number of students	40	12	9	7	4	72

4.3 Chance and Probability

Sometimes it happens that during rainy season, you carry a raincoat every day and it does not rain for many days. However, by chance, one day you forget to take the raincoat and it rains heavily on that day.

Sometimes it so happens that a student prepares 4 chapters out of 5, very well for a test. But a major question is asked from the chapter that she left unprepared.

Everyone knows that a particular train runs in time but the day you reach well in time it is late!

You face a lot of situations such as these where you take a chance and it does not go the way you want it to. Can you give some more examples? These are examples where the chances of a certain thing happening or not happening are not equal. The chances of the train being in time or being late are not the same. When you buy a ticket which is wait listed, you do take a chance. You hope that it might get confirmed by the time you travel.

We however, consider here certain experiments whose results have an equal chance of occurring.

4.3.1 Getting a result

You might have seen that before a cricket match starts, captains of the two teams go out to toss a coin to decide which team will bat first.

What are the possible results you get when a coin is tossed? Of course, Head or Tail.

Imagine that you are the captain of one team and your friend is the captain of the other team. You toss a coin and ask your friend to make the call. Can you control the result of the toss? Can you get a head if you want one? Or a tail if you want that? No, that is not possible. Such an experiment is called a **random experiment**. Head or Tail are the two **outcomes** of this experiment.

TRY THESE

1. If you try to start a scooter, what are the possible outcomes?
2. When a die is thrown, what are the six possible outcomes?

© NCERT not to be republished

3. When you spin the wheel shown, what are the possible outcomes? (Fig 4.6) List them.
(Outcome here means the sector at which the pointer stops).

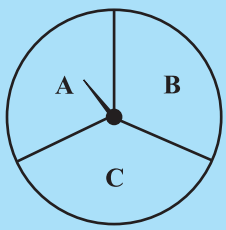


Fig 4.6

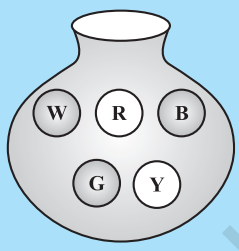


Fig 4.7

4. You have a bag with five identical balls of different colours and you are to pull out (draw) a ball without looking at it; list the outcomes you would get (Fig 4.7).

THINK, DISCUSS AND WRITE

In throwing a die:

- Does the first player have a greater chance of getting a six?
- Would the player who played after him have a lesser chance of getting a six?
- Suppose the second player got a six. Does it mean that the third player would not have a chance of getting a six?

4.3.2 Equally likely outcomes:

A coin is tossed several times and the number of times we get head or tail is noted. Let us look at the result sheet where we keep on increasing the tosses:

Number of tosses	Tally marks (H)	Number of heads	Tally mark (T)	Number of tails
50	𝍸 𝍸 𝍸 𝍸 𝍸 ||	27	𝍸 𝍸 𝍸 𝍸 |||	23
60	𝍸 𝍸 𝍸 𝍸 𝍸 |||	28	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	32
70	...	33	...	37
80	...	38	...	42
90	...	44	...	46
100	...	48	...	52

© NCERT not to be republished

Observe that as you increase the number of tosses more and more, the number of heads and the number of tails come closer and closer to each other.

This could also be done with a die, when tossed a large number of times. Number of each of the six outcomes become almost equal to each other.

In such cases, we may say that the different outcomes of the experiment are equally likely. This means that each of the outcomes has the same chance of occurring.

4.3.3 Linking chances to probability

Consider the experiment of tossing a coin once. What are the outcomes? There are only two outcomes – Head or Tail. Both the outcomes are equally likely. Likelihood of getting a head is one out of two outcomes, i.e., $\frac{1}{2}$. In other words, we say that the probability of getting a head = $\frac{1}{2}$. What is the probability of getting a tail?

Now take the example of throwing a die marked with 1, 2, 3, 4, 5, 6 on its faces (one number on one face). If you throw it once, what are the outcomes?

The outcomes are: 1, 2, 3, 4, 5, 6. Thus, there are six equally likely outcomes.

What is the probability of getting the outcome '2'?

It is $\frac{1 \leftarrow \text{Number of outcomes giving 2}}{6 \leftarrow \text{Number of equally likely outcomes.}}$

What is the probability of getting the number 5? What is the probability of getting the number 7? What is the probability of getting a number 1 through 6?

4.3.4 Outcomes as events

Each outcome of an experiment or a collection of outcomes make an **event**.

For example in the experiment of tossing a coin, getting a Head is an event and getting a Tail is also an event.

In case of throwing a die, getting each of the outcomes 1, 2, 3, 4, 5 or 6 is an event.

© NCERT not to be republished

Is getting an even number an event? Since an even number could be 2, 4 or 6, getting an even number is also an event. What will be the probability of getting an even number?

It is $\frac{3}{6}$ ← Number of outcomes that make the event
← Total number of outcomes of the experiment.

Example 3: A bag has 4 red balls and 2 yellow balls. (The balls are identical in all respects other than colour). A ball is drawn from the bag without looking into the bag. What is probability of getting a red ball? Is it more or less than getting a yellow ball?

Solution: There are in all (4 + 2 =) 6 outcomes of the event. Getting a red ball consists of 4 outcomes. (Why?)

Therefore, the probability of getting a red ball is $\frac{4}{6} = \frac{2}{3}$. In the same way the probability of getting a yellow ball = $\frac{2}{6} = \frac{1}{3}$ (Why?). Therefore, the probability of getting a red ball is more than that of getting a yellow ball.

TRY THESE

Suppose you spin the wheel

1. (i) List the number of outcomes of getting a green sector and not getting a green sector on this wheel (Fig 4.8).
   (ii) Find the probability of getting a green sector.
   (iii) Find the probability of not getting a green sector.

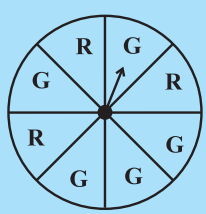


Fig 4.8

4.3.5 Chance and probability related to real life

We talked about the chance that it rains just on the day when we do not carry a rain coat.

What could you say about the chance in terms of probability? Could it be one in 10 days during a rainy season? The probability that it rains is then $\frac{1}{10}$. The probability that it does not rain = $\frac{9}{10}$. (Assuming raining or not raining on a day are equally likely)

The use of probability is made in various cases in real life.

1. To find characteristics of a large group by using a small part of the group.

   For example, during elections 'an exit poll' is taken. This involves asking the people whom they have voted for, when they come out after voting at the centres which are chosen off hand and distributed over the whole area. This gives an idea of chance of winning of each candidate and predictions are made based on it accordingly.

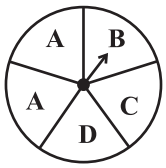
© NCERT not to be republished

2. Meteorological Department predicts weather by observing trends from the data over many years in the past.

EXERCISE 4.2

1. List the outcomes you can see in these experiments.

(a) Spinning a wheel



(b) Tossing two coins together

2. When a die is thrown, list the outcomes of an event of getting

(i) (a) a prime number (b) not a prime number.

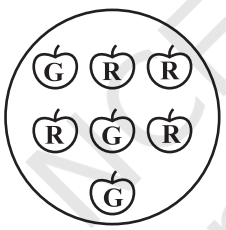
(ii) (a) a number greater than 5 (b) a number not greater than 5.

3. Find the.

(a) Probability of the pointer stopping on D in (Question 1-(a))?

(b) Probability of getting an ace from a well shuffled deck of 52 playing cards?

(c) Probability of getting a red apple. (See figure below)



4. Numbers 1 to 10 are written on ten separate slips (one number on one slip), kept in a box and mixed well. One slip is chosen from the box without looking into it. What is the probability of .

(i) getting a number 6?

(ii) getting a number less than 6?

(iii) getting a number greater than 6?

(iv) getting a 1-digit number?

5. If you have a spinning wheel with 3 green sectors, 1 blue sector and 1 red sector, what is the probability of getting a green sector? What is the probability of getting a non blue sector?

6. Find the probabilities of the events given in Question 2.

WHAT HAVE WE DISCUSSED?

1. In order to draw meaningful inferences from any data, we need to organise the data systematically.

2. Data can also be presented using **circle graph** or **pie chart**. A circle graph shows the relationship between a whole and its part.

© NCERT not to be republished

3. There are certain experiments whose outcomes have an equal chance of occurring.
4. A **random experiment** is one whose outcome cannot be predicted exactly in advance.
5. Outcomes of an experiment are **equally likely** if each has the same chance of occurring.
6. **Probability of an event** $= \frac{\text{Number of outcomes that make an event}}{\text{Total number of outcomes of the experiment}}$, when the outcomes are equally likely.
7. One or more outcomes of an experiment make an **event**.
8. Chances and probability are related to real life.

© NCERT not to be republished

CHAPTER

5

Squares and Square Roots

0852CH06

5.1 Introduction

You know that the area of a square = side × side (where 'side' means 'the length of a side'). Study the following table.

Side of a square (in cm)	Area of the square (in cm^2)
1	$1 \times 1 = 1 = 1^2$
2	$2 \times 2 = 4 = 2^2$
3	$3 \times 3 = 9 = 3^2$
5	$5 \times 5 = 25 = 5^2$
8	$8 \times 8 = 64 = 8^2$
a	$a \times a = a^2$

What is special about the numbers 4, 9, 25, 64 and other such numbers?

Since, 4 can be expressed as $2 \times 2 = 2^2$, 9 can be expressed as $3 \times 3 = 3^2$, all such numbers can be expressed as the product of the number with itself.

Such numbers like 1, 4, 9, 16, 25, ... are known as **square numbers**.

In general, if a natural number m can be expressed as n^2, where n is also a natural number, then m is a **square number**. Is 32 a square number?

We know that $5^2 = 25$ and $6^2 = 36$. If 32 is a square number, it must be the square of a natural number between 5 and 6. But there is no natural number between 5 and 6.

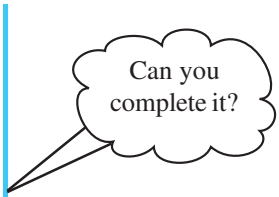
Therefore 32 is not a square number.

Consider the following numbers and their squares.

Number	Square
1	$1 \times 1 = 1$
2	$2 \times 2 = 4$

© NCERT not to be republished

3	$3 \times 3 = 9$
4	$4 \times 4 = 16$
5	$5 \times 5 = 25$
6	----------
7	----------
8	----------
9	----------
10	----------



From the above table, can we enlist the square numbers between 1 and 100? Are there any natural square numbers upto 100 left out?
You will find that the rest of the numbers are not square numbers.

The numbers 1, 4, 9, 16 ... are square numbers. These numbers are also called **perfect squares**.

TRY THESE

1. Find the perfect square numbers between (i) 30 and 40 (ii) 50 and 60

5.2 Properties of Square Numbers

Following table shows the squares of numbers from 1 to 20.

Number	Square	Number	Square
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

Study the square numbers in the above table. What are the ending digits (that is, digits in the units place) of the square numbers? All these numbers end with 0, 1, 4, 5, 6 or 9 at units place. None of these end with 2, 3, 7 or 8 at unit's place.

Can we say that if a number ends in 0, 1, 4, 5, 6 or 9, then it must be a square number? Think about it.

TRY THESE

1. Can we say whether the following numbers are perfect squares? How do we know?
   (i) 1057 (ii) 23453 (iii) 7928 (iv) 222222
   (v) 1069 (vi) 2061

© NCERT not to be republished

Write five numbers which you can decide by looking at their units digit that they are not square numbers.

2. Write five numbers which you cannot decide just by looking at their units digit (or units place) whether they are square numbers or not.

- Study the following table of some numbers and their squares and observe the one's place in both.

Table 1

Number	Square	Number	Square	Number	Square
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	30	900
7	49	17	289	35	1225
8	64	18	324	40	1600
9	81	19	361	45	2025
10	100	20	400	50	2500

The following square numbers end with digit 1.

Square	Number
1	1
81	9
121	11
361	19
441	21

TRY THESE

Which of 123^2, 77^2, 82^2, 161^2, 109^2 would end with digit 1?

Write the next two square numbers which end in 1 and their corresponding numbers.
You will see that if a number has 1 or 9 in the units place, then it's square ends in 1.

- Let us consider square numbers ending in 6.

Square	Number
16	4
36	6
196	14
256	16

TRY THESE

Which of the following numbers would have digit 6 at unit place.

(i) 19^2 (ii) 24^2 (iii) 26^2
(iv) 36^2 (v) 34^2

© NCERT not to be republished

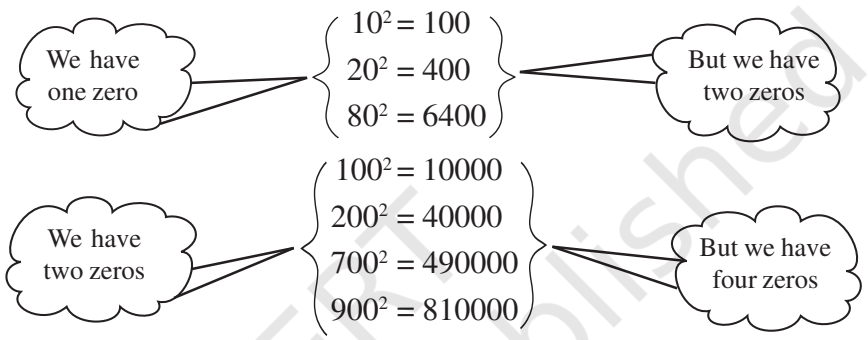
We can see that *when a square number ends in 6, the number whose square it is, will have either 4 or 6 in unit's place*.
Can you find more such rules by observing the numbers and their squares (Table 1)?

TRY THESE

What will be the "one's digit" in the square of the following numbers?

(i) 1234 (ii) 26387 (iii) 52698 (iv) 99880
(v) 21222 (vi) 9106

- Consider the following numbers and their squares.



If a number contains 3 zeros at the end, how many zeros will its square have ?
What do you notice about the number of zeros at the end of the number and the number of zeros at the end of its square?
Can we say that square numbers can only have even number of zeros at the end?

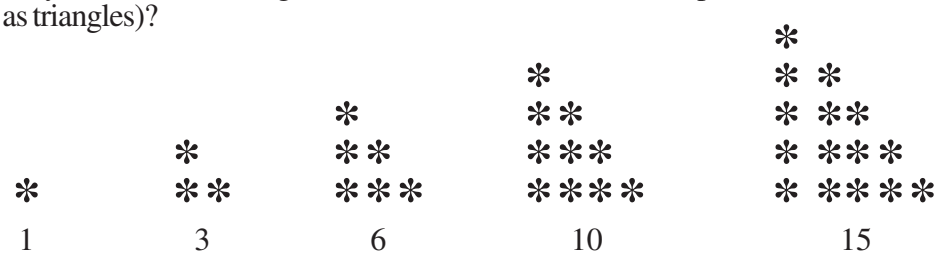
- See Table 1 with numbers and their squares.
What can you say about the squares of even numbers and squares of odd numbers?

TRY THESE

1. The square of which of the following numbers would be an odd number/an even number? Why?
(i) 727 (ii) 158 (iii) 269 (iv) 1980
2. What will be the number of zeros in the square of the following numbers?
(i) 60 (ii) 400

5.3 Some More Interesting Patterns

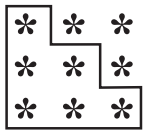
1. Adding triangular numbers.
Do you remember triangular numbers (numbers whose dot patterns can be arranged as triangles)?



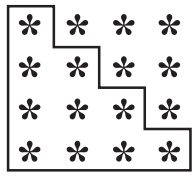
© NCERT not to be republished

If we combine two consecutive triangular numbers, we get a square number, like

$1 + 3 = 4$
$= 2^2$



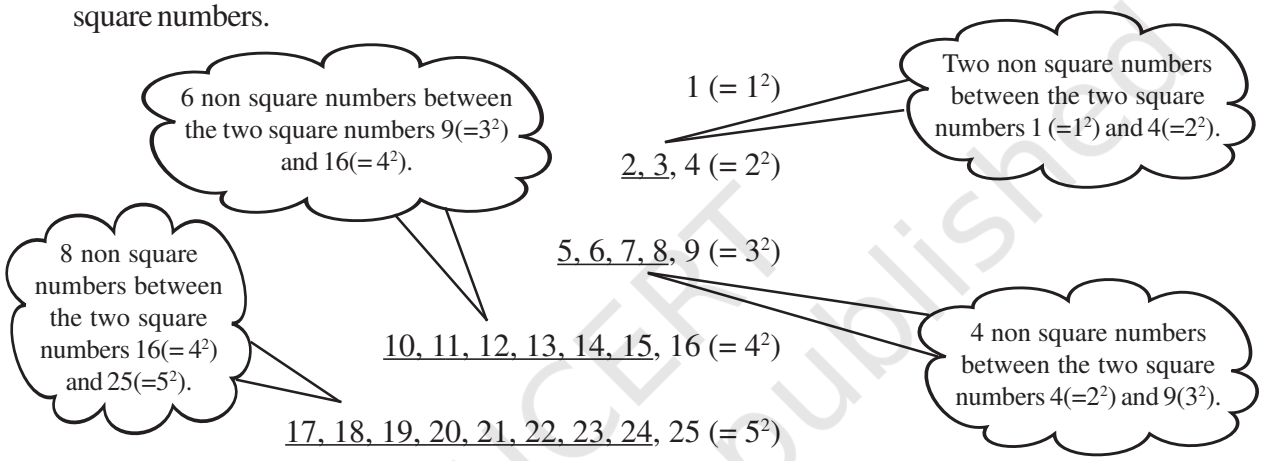
$3 + 6 = 9$
$= 3^2$



$6 + 10 = 16$
$= 4^2$

2. Numbers between square numbers

Let us now see if we can find some interesting pattern between two consecutive square numbers.



Between $1^2(=1)$ and $2^2(=4)$ there are two (i.e., 2×1) non square numbers 2, 3.

Between $2^2(=4)$ and $3^2(=9)$ there are four (i.e., 2×2) non square numbers 5, 6, 7, 8.

Now, $3^2 = 9$, $4^2 = 16$

Therefore, $4^2 - 3^2 = 16 - 9 = 7$

Between $9(=3^2)$ and $16(=4^2)$ the numbers are 10, 11, 12, 13, 14, 15 that is, six non-square numbers which is 1 less than the difference of two squares.

We have $4^2 = 16$ and $5^2 = 25$

Therefore, $5^2 - 4^2 = 9$

Between $16(=4^2)$ and $25(=5^2)$ the numbers are 17, 18, ... , 24 that is, eight non square numbers which is 1 less than the difference of two squares.

Consider 7^2 and 6^2. Can you say how many numbers are there between 6^2 and 7^2? If we think of any natural number n and $(n + 1)$, then,

$$(n + 1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1.$$

We find that between n^2 and $(n + 1)^2$ there are $2n$ numbers which is 1 less than the difference of two squares.

Thus, in general we can say that *there are 2n non perfect square numbers between the squares of the numbers n and* $(n + 1)$. Check for $n = 5$, $n = 6$ etc., and verify.

© NCERT not to be republished

TRY THESE

1. How many natural numbers lie between 9^2 and 10^2 ? Between 11^2 and 12^2?
2. How many non square numbers lie between the following pairs of numbers
   (i) 100^2 and 101^2 (ii) 90^2 and 91^2 (iii) 1000^2 and 1001^2

3. Adding odd numbers

Consider the following

1 [one odd number]	$= 1 = 1^2$
1 + 3 [sum of first two odd numbers]	$= 4 = 2^2$
1 + 3 + 5 [sum of first three odd numbers]	$= 9 = 3^2$
1 + 3 + 5 + 7 [...]	$= 16 = 4^2$
1 + 3 + 5 + 7 + 9 [...]	$= 25 = 5^2$
1 + 3 + 5 + 7 + 9 + 11 [...]	$= 36 = 6^2$

So we can say that the *sum of first n odd natural numbers is* n^2.

Looking at it in a different way, we can say: 'If the number is a square number, it has to be the sum of successive **odd** numbers starting from 1.

Consider those numbers which are not perfect squares, say 2, 3, 5, 6, ... Can you express these numbers as a sum of successive odd natural numbers beginning from 1?

You will find that these numbers cannot be expressed in this form.

Consider the number 25. Successively subtract 1, 3, 5, 7, 9, ... from it

(i) $25 - 1 = 24$ (ii) $24 - 3 = 21$ (iii) $21 - 5 = 16$ (iv) $16 - 7 = 9$

(v) $9 - 9 = 0$

This means, $25 = 1 + 3 + 5 + 7 + 9$. Also, 25 is a perfect square.

Now consider another number 38, and again do as above.

(i) $38 - 1 = 37$ (ii) $37 - 3 = 34$ (iii) $34 - 5 = 29$ (iv) $29 - 7 = 22$

(v) $22 - 9 = 13$ (vi) $13 - 11 = 2$ (vii) $2 - 13 = -11$

This shows that we are not able to express 38 as the sum of consecutive odd numbers starting with 1. Also, 38 is not a perfect square.

So we can also say that *if a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.*

We can use this result to find whether a number is a perfect square or not.

TRY THESE

Find whether each of the following numbers is a perfect square or not?

(i) 121 (ii) 55 (iii) 81

(iv) 49 (v) 69

4. A sum of consecutive natural numbers

Consider the following

$3^2 = 9 = 4 + 5$

$5^2 = 25 = 12 + 13$

$7^2 = 49 = 24 + 25$

First Number $= \frac{3^2 - 1}{2}$

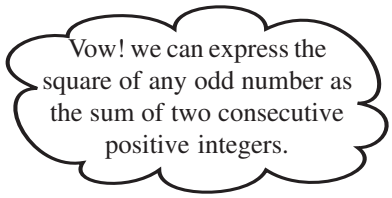
Second Number $= \frac{3^2 + 1}{2}$

© NCERT not to be republished

$9^2 = 81 = 40 + 41$

$11^2 = 121 = 60 + 61$

$15^2 = 225 = 112 + 113$



TRY THESE

1. Express the following as the sum of two consecutive integers.
   (i) 21^2 (ii) 13^2 (iii) 11^2 (iv) 19^2
2. Do you think the reverse is also true, i.e., is the sum of any two consecutive positive integers is perfect square of a number? Give example to support your answer.

5. Product of two consecutive even or odd natural numbers

$11 \times 13 = 143 = 12^2 - 1$

Also $11 \times 13 = (12 - 1) \times (12 + 1)$

Therefore, $11 \times 13 = (12 - 1) \times (12 + 1) = 12^2 - 1$

Similarly, $13 \times 15 = (14 - 1) \times (14 + 1) = 14^2 - 1$

$29 \times 31 = (30 - 1) \times (30 + 1) = 30^2 - 1$

$44 \times 46 = (45 - 1) \times (45 + 1) = 45^2 - 1$

So in general we can say that $(a + 1) \times (a - 1) = a^2 - 1$.

6. Some more patterns in square numbers

Observe the squares of numbers; 1, 11, 111 ... etc. They give a beautiful pattern:

$1^2 = 1$

$11^2 = 1\ 2\ 1$

$111^2 = 1\ 2\ 3\ 2\ 1$

$1111^2 = 1\ 2\ 3\ 4\ 3\ 2\ 1$

$11111^2 = 1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$

$1111111^2 = 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1$

Another interesting pattern.

$7^2 = 49$

$67^2 = 4489$

$667^2 = 444889$

$6667^2 = 44448889$

$66667^2 = 4444488889$

$666667^2 = 444444888889$

The fun is in being able to find out why this happens. May be it would be interesting for you to explore and think about such questions even if the answers come some years later.

TRY THESE

Write the square, making use of the above pattern.
(i) 111111^2 (ii) 1111111^2

TRY THESE

Can you find the square of the following numbers using the above pattern?
(i) 6666667^2 (ii) 66666667^2

© NCERT not to be republished

EXERCISE 5.1

1. What will be the unit digit of the squares of the following numbers?

(i) 81 (ii) 272 (iii) 799 (iv) 3853

(v) 1234 (vi) 26387 (vii) 52698 (viii) 99880

(ix) 12796 (x) 55555

2. The following numbers are obviously not perfect squares. Give reason.

(i) 1057 (ii) 23453 (iii) 7928 (iv) 222222

(v) 64000 (vi) 89722 (vii) 222000 (viii) 505050

3. The squares of which of the following would be odd numbers?

(i) 431 (ii) 2826 (iii) 7779 (iv) 82004

4. Observe the following pattern and find the missing digits.

$11^2 = 121$

$101^2 = 10201$

$1001^2 = 1002001$

$100001^2 = 1$ 2 1

$10000001^2 =$ .....

5. Observe the following pattern and supply the missing numbers.

$11^2 = 1\ 2\ 1$

$101^2 = 1\ 0\ 2\ 0\ 1$

$10101^2 = 102030201$

$1010101^2 =$ .....

.....$^2 = 10203040504030201$

6. Using the given pattern, find the missing numbers.

$1^2 + 2^2 + 2^2 = 3^2$

$2^2 + 3^2 + 6^2 = 7^2$

$3^2 + 4^2 + 12^2 = 13^2$

$4^2 + 5^2 + \_^2 = 21^2$

$5^2 + \_^2 + 30^2 = 31^2$

$6^2 + 7^2 + \_^2 = \_\_^2$

To find pattern

Third number is related to first and second number. How?

Fourth number is related to third number. How?

7. Without adding, find the sum.

(i) 1 + 3 + 5 + 7 + 9

(ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 +19

(iii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23

8. (i) Express 49 as the sum of 7 odd numbers.

(ii) Express 121 as the sum of 11 odd numbers.

9. How many numbers lie between squares of the following numbers?

(i) 12 and 13 (ii) 25 and 26 (iii) 99 and 100

© NCERT not to be republished

5.4 Finding the Square of a Number

Squares of small numbers like 3, 4, 5, 6, 7, ... etc. are easy to find. But can we find the square of 23 so quickly?

The answer is not so easy and we may need to multiply 23 by 23.

There is a way to find this without having to multiply 23×23.

We know $23 = 20 + 3$

Therefore $23^2 = (20 + 3)^2 = 20(20 + 3) + 3(20 + 3)$

$= 20^2 + 20 \times 3 + 3 \times 20 + 3^2$

$= 400 + 60 + 60 + 9 = 529$

Example 1: Find the square of the following numbers without actual multiplication.

(i) 39 (ii) 42

Solution: (i) $39^2 = (30 + 9)^2 = 30(30 + 9) + 9(30 + 9)$

$= 30^2 + 30 \times 9 + 9 \times 30 + 9^2$

$= 900 + 270 + 270 + 81 = 1521$

(ii) $42^2 = (40 + 2)^2 = 40(40 + 2) + 2(40 + 2)$

$= 40^2 + 40 \times 2 + 2 \times 40 + 2^2$

$= 1600 + 80 + 80 + 4 = 1764$

5.4.1 Other patterns in squares

Consider the following pattern:

$25^2 = 625 = (2 \times 3)$ hundreds + 25

$35^2 = 1225 = (3 \times 4)$ hundreds + 25

$75^2 = 5625 = (7 \times 8)$ hundreds + 25

$125^2 = 15625 = (12 \times 13)$ hundreds + 25

Now can you find the square of 95?

Consider a number with unit digit 5, i.e., $a5$

$(a5)^2 = (10a + 5)^2$

$= 10a(10a + 5) + 5(10a + 5)$

$= 100a^2 + 50a + 50a + 25$

$= 100a(a + 1) + 25$

$= a(a + 1)$ hundred + 25

TRY THESE

Find the squares of the following numbers containing 5 in unit's place.

(i) 15 (ii) 95 (iii) 105 (iv) 205

5.4.2 Pythagorean triplets

Consider the following

$3^2 + 4^2 = 9 + 16 = 25 = 5^2$

The collection of numbers 3, 4 and 5 is known as **Pythagorean triplet**. 6, 8, 10 is also a Pythagorean triplet, since

$6^2 + 8^2 = 36 + 64 = 100 = 10^2$

Again, observe that

$5^2 + 12^2 = 25 + 144 = 169 = 13^2$. The numbers 5, 12, 13 form another such triplet.

© NCERT not to be republished

Can you find more such triplets?

For any natural number $m > 1$, we have $(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$. So, $2m$, $m^2 - 1$ and $m^2 + 1$ forms a Pythagorean triplet.

Try to find some more Pythagorean triplets using this form.

Example 2: Write a Pythagorean triplet whose smallest member is 8.

Solution: We can get Pythagorean triplets by using general form $2m, m^2 - 1, m^2 + 1$.

Let us first take $m^2 - 1 = 8$

So, $m^2 = 8 + 1 = 9$

which gives $m = 3$

Therefore, $2m = 6$ and $m^2 + 1 = 10$

The triplet is thus 6, 8, 10. But 8 is not the smallest member of this.

So, let us try $2m = 8$

then $m = 4$

We get $m^2 - 1 = 16 - 1 = 15$

and $m^2 + 1 = 16 + 1 = 17$

The triplet is 8, 15, 17 with 8 as the smallest member.

Example 3: Find a Pythagorean triplet in which one member is 12.

Solution: If we take $m^2 - 1 = 12$

Then, $m^2 = 12 + 1 = 13$

Then the value of m will not be an integer.

So, we try to take $m^2 + 1 = 12$. Again $m^2 = 11$ will not give an integer value for m.

So, let us take $2m = 12$

then $m = 6$

Thus, $m^2 - 1 = 36 - 1 = 35$ and $m^2 + 1 = 36 + 1 = 37$

Therefore, the required triplet is 12, 35, 37.

Note: All Pythagorean triplets may not be obtained using this form. For example another triplet 5, 12, 13 also has 12 as a member.

EXERCISE 5.2

1. Find the square of the following numbers.

(i) 32 (ii) 35 (iii) 86 (iv) 93

(v) 71 (vi) 46

2. Write a Pythagorean triplet whose one member is.

(i) 6 (ii) 14 (iii) 16 (iv) 18

5.5 Square Roots

Study the following situations.

(a) Area of a square is 144 cm^2. What could be the side of the square?

© NCERT not to be republished

We know that the area of a square = side^2

If we assume the length of the side to be 'a', then $144 = a^2$

To find the length of side it is necessary to find a number whose square is 144.

(b) What is the length of a diagonal of a square of side 8 cm (Fig 5.1)?

Can we use Pythagoras theorem to solve this ?

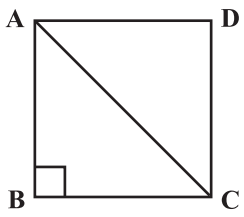


Fig 5.1

We have, $AB^2 + BC^2 = AC^2$

i.e., $8^2 + 8^2 = AC^2$

or $64 + 64 = AC^2$

or $128 = AC^2$

Again to get AC we need to think of a number whose square is 128.

(c) In a right triangle the length of the hypotenuse and a side are respectively 5 cm and 3 cm (Fig 5.2).

Can you find the third side?

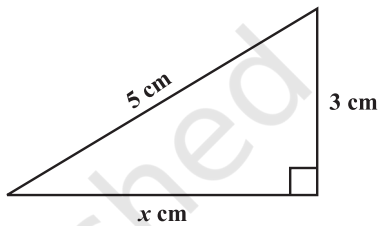


Fig 5.2

Let x cm be the length of the third side.

Using Pythagoras theorem $5^2 = x^2 + 3^2$

$25 - 9 = x^2$

$16 = x^2$

Again, to find x we need a number whose square is 16.

In all the above cases, we need to find a number whose square is known. Finding the number with the known square is known as finding the square root.

5.5.1 Finding square roots

The inverse (opposite) operation of addition is subtraction and the inverse operation of multiplication is division. Similarly, finding the square root is the inverse operation of squaring.

We have, $1^2 = 1$, therefore square root of 1 is 1

$2^2 = 4$, therefore square root of 4 is 2

$3^2 = 9$, therefore square root of 9 is 3

Since $9^2 = 81$,
and $(-9)^2 = 81$
We say that square roots of 81 are 9 and –9.

TRY THESE

(i) $11^2 = 121$. What is the square root of 121?

(ii) $14^2 = 196$. What is the square root of 196?

THINK, DISCUSS AND WRITE

$(-1)^2 = 1$. Is –1, a square root of 1? $(-2)^2 = 4$. Is –2, a square root of 4?

$(-9)^2 = 81$. Is –9 a square root of 81?

From the above, you may say that there are two integral square roots of a perfect square number. In this chapter, we shall take up only positive square root of a natural number.

Positive square root of a number is denoted by the symbol $\sqrt{\ }$.

For example: $\sqrt{4} = 2$ (not –2); $\sqrt{9} = 3$ (not –3) etc.

© NCERT not to be republished

Statement	Inference
$1^2 = 1$	$\sqrt{1} = 1$
$2^2 = 4$	$\sqrt{4} = 2$
$3^2 = 9$	$\sqrt{9} = 3$
$4^2 = 16$	$\sqrt{16} = 4$
$5^2 = 25$	$\sqrt{25} = 5$

Statement	Inference
$6^2 = 36$	$\sqrt{36} = 6$
$7^2 = 49$	$\sqrt{49} = 7$
$8^2 = 64$	$\sqrt{64} = 8$
$9^2 = 81$	$\sqrt{81} = 9$
$10^2 = 100$	$\sqrt{100} = 10$

5.5.2 Finding square root through repeated subtraction

Do you remember that the sum of the first n odd natural numbers is n^2? That is, every square number can be expressed as a sum of successive odd natural numbers starting from 1.

Consider $\sqrt{81}$. Then,

(i) $81 - 1 = 80$ (ii) $80 - 3 = 77$ (iii) $77 - 5 = 72$ (iv) $72 - 7 = 65$
(v) $65 - 9 = 56$ (vi) $56 - 11 = 45$ (vii) $45 - 13 = 32$ (viii) $32 - 15 = 17$
(ix) $17 - 17 = 0$

From 81 we have subtracted successive odd numbers starting from 1 and obtained 0 at 9th step.

Therefore $\sqrt{81} = 9$.

Can you find the square root of 729 using this method? Yes, but it will be time consuming. Let us try to find it in a simpler way.

TRY THESE

By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect squares or not? If the number is a perfect square then find its square root.

(i) 121
(ii) 55
(iii) 36
(iv) 49
(v) 90

5.5.3 Finding square root through prime factorisation

Consider the prime factorisation of the following numbers and their squares.

Prime factorisation of a Number	Prime factorisation of its Square
$6 = 2 \times 3$	$36 = 2 \times 2 \times 3 \times 3$
$8 = 2 \times 2 \times 2$	$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$12 = 2 \times 2 \times 3$	$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$15 = 3 \times 5$	$225 = 3 \times 3 \times 5 \times 5$

How many times does 2 occur in the prime factorisation of 6? Once. How many times does 2 occur in the prime factorisation of 36? Twice. Similarly, observe the occurrence of 3 in 6 and 36 of 2 in 8 and 64 etc.

You will find that each prime factor in the prime factorisation of the square of a number, occurs twice the number of times it occurs in the prime factorisation of the number itself. Let us use this to find the square root of a given square number, say 324.

We know that the prime factorisation of 324 is

$$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$$

2	324
2	162
3	81
3	27
3	9
	3

© NCERT not to be republished

By pairing the prime factors, we get

$$324 = \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} = 2^2 \times 3^2 \times 3^2 = (2 \times 3 \times 3)^2$$

So, $\sqrt{324} = 2 \times 3 \times 3 = 18$

Similarly can you find the square root of 256? Prime factorisation of 256 is

$$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

By pairing the prime factors we get,

$$256 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} = (2 \times 2 \times 2 \times 2)^2$$

Therefore, $\sqrt{256} = 2 \times 2 \times 2 \times 2 = 16$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

Is 48 a perfect square?

We know $48 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3$

Since all the factors are not in pairs so 48 is not a perfect square.

Suppose we want to find the smallest multiple of 48 that is a perfect square, how should we proceed? Making pairs of the prime factors of 48 we see that 3 is the only factor that does not have a pair. So we need to multiply by 3 to complete the pair.

Hence $48 \times 3 = 144$ is a perfect square.

Can you tell by which number should we divide 48 to get a perfect square?

The factor 3 is not in pair, so if we divide 48 by 3 we get $48 \div 3 = 16 = \underline{2 \times 2} \times \underline{2 \times 2}$ and this number 16 is a perfect square too.

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
5	25
	5

Example 4: Find the square root of 6400.

Solution: Write $6400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$

Therefore $\sqrt{6400} = 2 \times 2 \times 2 \times 2 \times 5 = 80$

Example 5: Is 90 a perfect square?

2	90
3	45
3	15
	5

Solution: We have $90 = 2 \times 3 \times 3 \times 5$

The prime factors 2 and 5 do not occur in pairs. Therefore, 90 is not a perfect square. That 90 is not a perfect square can also be seen from the fact that it has only one zero.

Example 6: Is 2352 a perfect square? If not, find the smallest multiple of 2352 which is a perfect square. Find the square root of the new number.

2	2352
2	1176
2	588
2	294
3	147
7	49
	7

Solution: We have $2352 = \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$

As the prime factor 3 has no pair, 2352 is not a perfect square.

If 3 gets a pair then the number will become perfect square. So, we multiply 2352 by 3 to get,

$$2352 \times 3 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{7 \times 7}$$

Now each prime factor is in a pair. Therefore, $2352 \times 3 = 7056$ is a perfect square. Thus the required smallest multiple of 2352 is 7056 which is a perfect square.

And, $\sqrt{7056} = 2 \times 2 \times 3 \times 7 = 84$

Example 7: Find the smallest number by which 9408 must be divided so that the quotient is a perfect square. Find the square root of the quotient.

© NCERT not to be republished

Solution: We have, $9408 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$
If we divide 9408 by the factor 3, then
$9408 \div 3 = 3136 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$ which is a perfect square. (Why?)
Therefore, the required smallest number is 3.

And, $\sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$.

2	6, 9, 15
3	3, 9, 15
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

Example 8: Find the smallest square number which is divisible by each of the numbers 6, 9 and 15.

Solution: This has to be done in two steps. First find the smallest common multiple and then find the square number needed. The least number divisible by each one of 6, 9 and 15 is their LCM. The LCM of 6, 9 and 15 is $2 \times 3 \times 3 \times 5 = 90$.
Prime factorisation of 90 is $90 = 2 \times \underline{3 \times 3} \times 5$.

We see that prime factors 2 and 5 are not in pairs. Therefore 90 is not a perfect square.

In order to get a perfect square, each factor of 90 must be paired. So we need to make pairs of 2 and 5. Therefore, 90 should be multiplied by 2×5, i.e., 10.
Hence, the required square number is $90 \times 10 = 900$.

EXERCISE 5.3

1. What could be the possible 'one's' digits of the square root of each of the following numbers?
(i) 9801 (ii) 99856 (iii) 998001 (iv) 657666025

2. Without doing any calculation, find the numbers which are surely not perfect squares.
(i) 153 (ii) 257 (iii) 408 (iv) 441

3. Find the square roots of 100 and 169 by the method of repeated subtraction.

4. Find the square roots of the following numbers by the Prime Factorisation Method.
(i) 729 (ii) 400 (iii) 1764 (iv) 4096
(v) 7744 (vi) 9604 (vii) 5929 (viii) 9216
(ix) 529 (x) 8100

5. For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.
(i) 252 (ii) 180 (iii) 1008 (iv) 2028
(v) 1458 (vi) 768

6. For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also find the square root of the square number so obtained.
(i) 252 (ii) 2925 (iii) 396 (iv) 2645
(v) 2800 (vi) 1620

7. The students of Class VIII of a school donated ₹ 2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

© NCERT not to be republished

8. 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

9. Find the smallest square number that is divisible by each of the numbers 4, 9 and 10.

10. Find the smallest square number that is divisible by each of the numbers 8, 15 and 20.

5.5.4 Finding square root by division method

When the numbers are large, even the method of finding square root by prime factorisation becomes lengthy and difficult. To overcome this problem we use Long Division Method.

For this we need to determine the number of digits in the square root.
See the following table:

Number	Square	
10	100	which is the smallest 3-digit perfect square
31	961	which is the greatest 3-digit perfect square
32	1024	which is the smallest 4-digit perfect square
99	9801	which is the greatest 4-digit perfect square

So, what can we say about the number of digits in the square root if a perfect square is a 3-digit or a 4-digit number? We can say that, if a perfect square is a 3-digit or a 4-digit number, then its square root will have 2-digits.

Can you tell the number of digits in the square root of a 5-digit or a 6-digit perfect square?

The smallest 3-digit perfect square number is 100 which is the square of 10 and the greatest 3-digit perfect square number is 961 which is the square of 31. The smallest 4-digit square number is 1024 which is the square of 32 and the greatest 4-digit number is 9801 which is the square of 99.

THINK, DISCUSS AND WRITE

Can we say that if a perfect square is of n-digits, then its square root will have $\frac{n}{2}$ digits if n is even or $\frac{(n+1)}{2}$ if n is odd?

The use of the number of digits in square root of a number is useful in the following method:

- Consider the following steps to find the square root of 529.

Can you estimate the number of digits in the square root of this number?

Step 1 Place a bar over every pair of digits starting from the digit at one's place. If the number of digits in it is odd, then the left-most single digit too will have a bar.

Thus we have, $\overline{5}\,\overline{29}$.

Step 2 Find the largest number whose square is less than or equal to the number under the extreme left bar ($2^2 < 5 < 3^2$). Take this number as the divisor and the quotient with the number under the extreme left bar as the dividend (here 5). Divide and get the remainder (1 in this case).

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{5}\,\overline{29} \\ & -4 \\ \hline & 1 \end{array}$$

© NCERT not to be republished

$$\begin{array}{r|l} & \;\;2 \\ \hline 2 & \;\overline{5}\,\overline{29} \\ & -4 \\ \hline & \;\;1\,29 \end{array}$$

Step 3 Bring down the number under the next bar (i.e., 29 in this case) to the right of the remainder. So the new dividend is 129.

Step 4 Double the quotient and enter it with a blank on its right.

$$\begin{array}{r|l} & \;\;2 \\ \hline 2 & \;\overline{5}\,\overline{29} \\ & -4 \\ \hline 4\_ & \;\;1\,29 \end{array}$$

Step 5 Guess a largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied to the new quotient the product is less than or equal to the dividend.

In this case $42 \times 2 = 84$.

As $43 \times 3 = 129$ so we choose the new digit as 3. Get the remainder.

$$\begin{array}{r|l} & \;\;23 \\ \hline 2 & \;\overline{5}\,\overline{29} \\ & -4 \\ \hline 43 & \;\;1\,29 \\ & -129 \\ \hline & \;\;\;\;0 \end{array}$$

Step 6 Since the remainder is 0 and no digits are left in the given number, therefore, $\sqrt{529} = 23$.

- Now consider $\sqrt{4096}$

Step 1 Place a bar over every pair of digits starting from the one's digit. ($\overline{40}\,\overline{96}$).

$$\begin{array}{r|l} & \;\;6 \\ \hline 6 & \;\overline{40}\,\overline{96} \\ & -36 \\ \hline & \;\;\;4 \end{array}$$

Step 2 Find the largest number whose square is less than or equal to the number under the left-most bar ($6^2 < 40 < 7^2$). Take this number as the divisor and the number under the left-most bar as the dividend. Divide and get the remainder i.e., 4 in this case.

$$\begin{array}{r|l} & \;\;6 \\ \hline 6 & \;\overline{40}\,\overline{96} \\ & -36 \\ \hline & \;\;496 \end{array}$$

Step 3 Bring down the number under the next bar (i.e., 96) to the right of the remainder. The new dividend is 496.

$$\begin{array}{r|l} & \;\;6 \\ \hline 6 & \;\overline{40}\,\overline{96} \\ & -36 \\ \hline 12\_ & \;\;496 \end{array}$$

Step 4 Double the quotient and enter it with a blank on its right.

Step 5 Guess a largest possible digit to fill the blank which also becomes the new digit in the quotient such that when the new digit is multiplied to the new quotient the product is less than or equal to the dividend. In this case we see that $124 \times 4 = 496$.

So the new digit in the quotient is 4. Get the remainder.

$$\begin{array}{r|l} & \;\;6\,4 \\ \hline 6 & \;\overline{40}\,\overline{96} \\ & -36 \\ \hline 12\underline{4} & \;\;496 \\ & -496 \\ \hline & \;\;\;\;\;0 \end{array}$$

Step 6 Since the remainder is 0 and no bar left, therefore, $\sqrt{4096} = 64$.

Estimating the number

We use bars to find the number of digits in the square root of a perfect square number.

$$\sqrt{\overline{5}\,\overline{29}} = 23 \qquad \text{and} \qquad \sqrt{\overline{40}\,\overline{96}} = 64$$

In both the numbers 529 and 4096 there are two bars and the number of digits in their square root is 2. Can you tell the number of digits in the square root of 14400?

By placing bars we get $\overline{1}\,\overline{44}\,\overline{00}$. Since there are 3 bars, the square root will be of 3 digit.

© NCERT not to be republished

TRY THESE

Without calculating square roots, find the number of digits in the square root of the following numbers.

(i) 25600 (ii) 100000000 (iii) 36864

Example 9: Find the square root of : (i) 729 (ii) 1296

Solution:

(i)

	27
2	$\overline{7}\,\overline{29}$
	− 4
47	329
	329
	0

Therefore $\sqrt{729} = 27$

(ii)

	36
3	$\overline{12}\,\overline{96}$
	− 9
66	396
	396
	0

Therefore $\sqrt{1296} = 36$

Example 10: Find the least number that must be subtracted from 5607 so as to get a perfect square. Also find the square root of the perfect square.

Solution: Let us try to find $\sqrt{5607}$ by long division method. We get the remainder 131. It shows that 74^2 is less than 5607 by 131.

This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is 5607 – 131 = 5476. And, $\sqrt{5476} = 74$.

	74
7	$\overline{56}\,\overline{07}$
	− 49
144	707
	−576
	131

Example 11: Find the greatest 4-digit number which is a perfect square.

Solution: Greatest number of 4-digits = 9999. We find $\sqrt{9999}$ by long division method. The remainder is 198. This shows 99^2 is less than 9999 by 198.

This means if we subtract the remainder from the number, we get a perfect square.

Therefore, the required perfect square is 9999 – 198 = 9801.

And, $\sqrt{9801} = 99$

	99
9	$\overline{99}\,\overline{99}$
	− 81
189	1899
	− 1701
	198

Example 12: Find the least number that must be added to 1300 so as to get a perfect square. Also find the square root of the perfect square.

Solution: We find $\sqrt{1300}$ by long division method. The remainder is 4.

This shows that $36^2 < 1300$.

Next perfect square number is $37^2 = 1369$.

Hence, the number to be added is $37^2 - 1300 = 1369 - 1300 = 69$.

	36
3	$\overline{13}\,\overline{00}$
	− 9
66	400
	− 396
	4

5.6 Square Roots of Decimals

Consider $\sqrt{17.64}$

Step 1 To find the square root of a decimal number we put bars on the integral part (i.e., 17) of the number in the usual manner. And place bars on the decimal part

© NCERT not to be republished

(i.e., 64) on every pair of digits beginning with the first decimal place. Proceed as usual. We get $\overline{17}.\overline{64}$.

Step 2 Now proceed in a similar manner. The left most bar is on 17 and $4^2 < 17 < 5^2$. Take this number as the divisor and the number under the left-most bar as the dividend, i.e., 17. Divide and get the remainder.

Step 3 The remainder is 1. Write the number under the next bar (i.e., 64) to the right of this remainder, to get 164.

Step 4 Double the divisor and enter it with a blank on its right. Since 64 is the decimal part so put a decimal point in the quotient.

Step 5 We know $82 \times 2 = 164$, therefore, the new digit is 2. Divide and get the remainder.

Step 6 Since the remainder is 0 and no bar left, therefore $\sqrt{17.64} = 4.2$.

Example 13: Find the square root of 12.25.

Solution:

Therefore, $\sqrt{12.25} = 3.5$

Which way to move

Consider a number 176.341. Put bars on both integral part and decimal part. In what way is putting bars on decimal part different from integral part? Notice for 176 we start from the unit's place close to the decimal and move towards left. The first bar is over 76 and the second bar over 1. For .341, we start from the decimal and move towards right. First bar is over 34 and for the second bar we put 0 after 1 and make $.\overline{34}\overline{10}$.

Example 14: Area of a square plot is 2304 m^2. Find the side of the square plot.

Solution: Area of square plot = 2304 m^2

Therefore, side of the square plot = $\sqrt{2304}$ m

We find that, $\sqrt{2304} = 48$

Thus, the side of the square plot is 48 m.

Example 15: There are 2401 students in a school. P.T. teacher wants them to stand in rows and columns such that the number of rows is equal to the number of columns. Find the number of rows.

© NCERT not to be republished

Solution: Let the number of rows be x
So, the number of columns $= x$
Therefore, number of students $= x \times x = x^2$
Thus, $x^2 = 2401$ gives $x = \sqrt{2401} = 49$
The number of rows = 49.

	49
4	$\overline{24}\,\overline{01}$
	-16
89	801
	801
	0

EXERCISE 5.4

1. Find the square root of each of the following numbers by Division method.

(i) 2304 (ii) 4489 (iii) 3481 (iv) 529
(v) 3249 (vi) 1369 (vii) 5776 (viii) 7921
(ix) 576 (x) 1024 (xi) 3136 (xii) 900

2. Find the number of digits in the square root of each of the following numbers (without any calculation).

(i) 64 (ii) 144 (iii) 4489 (iv) 27225
(v) 390625

3. Find the square root of the following decimal numbers.

(i) 2.56 (ii) 7.29 (iii) 51.84 (iv) 42.25
(v) 31.36

4. Find the least number which must be subtracted from each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 402 (ii) 1989 (iii) 3250 (iv) 825
(v) 4000

5. Find the least number which must be added to each of the following numbers so as to get a perfect square. Also find the square root of the perfect square so obtained.

(i) 525 (ii) 1750 (iii) 252 (iv) 1825
(v) 6412

6. Find the length of the side of a square whose area is 441 m^2.

7. In a right triangle ABC, $\angle B = 90°$.

(a) If AB = 6 cm, BC = 8 cm, find AC (b) If AC = 13 cm, BC = 5 cm, find AB

8. A gardener has 1000 plants. He wants to plant these in such a way that the number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.

9. There are 500 children in a school. For a P.T. drill they have to stand in such a manner that the number of rows is equal to number of columns. How many children would be left out in this arrangement.

© NCERT not to be republished

WHAT HAVE WE DISCUSSED?

1. If a natural number m can be expressed as n^2, where n is also a natural number, then m is a **square number**.
2. All square numbers end with 0, 1, 4, 5, 6 or 9 at units place.
3. Square numbers can only have even number of zeros at the end.
4. **Square root** is the inverse operation of square.
5. There are two integral square roots of a perfect square number.

   Positive square root of a number is denoted by the symbol $\sqrt{\ }$.

   For example, $3^2 = 9$ gives $\sqrt{9} = 3$

© NCERT not to be republished

CHAPTER

6

Cubes and Cube Roots

0852CH07

6.1 Introduction

This is a story about one of India's great mathematical geniuses, S. Ramanujan. Once another famous mathematician Prof. G.H. Hardy came to visit him in a taxi whose number was 1729. While talking to Ramanujan, Hardy described this number "a dull number". Ramanujan quickly pointed out that 1729 was indeed interesting. He said it is the smallest number that can be expressed as a sum of two cubes in two different ways:

$$1729 = 1728 + 1 = 12^3 + 1^3$$

$$1729 = 1000 + 729 = 10^3 + 9^3$$

1729 has since been known as the Hardy – Ramanujan Number, even though this feature of 1729 was known more than 300 years before Ramanujan.

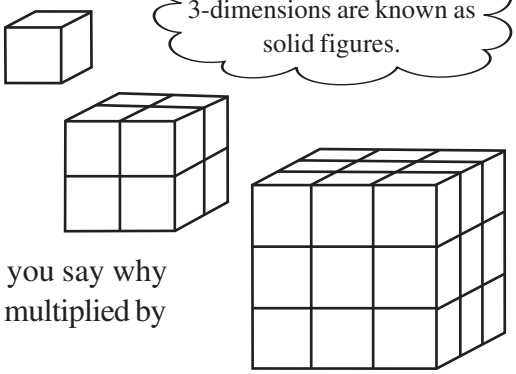
Hardy – Ramanujan Number

1729 is the smallest Hardy–Ramanujan Number. There are an infinitely many such numbers. Few are 4104 (2, 16; 9, 15), 13832 (18, 20; 2, 24), Check it with the numbers given in the brackets.

How did Ramanujan know this? Well, he loved numbers. All through his life, he experimented with numbers. He probably found numbers that were expressed as the sum of two squares and sum of two cubes also.

There are many other interesting patterns of cubes. Let us learn about cubes, cube roots and many other interesting facts related to them.

6.2 Cubes



You know that the word 'cube' is used in geometry. A cube is a solid figure which has all its sides equal. How many cubes of side 1 cm will make a cube of side 2 cm?

How many cubes of side 1 cm will make a cube of side 3 cm?

Consider the numbers 1, 8, 27, ...

These are called **perfect cubes or cube numbers**. Can you say why they are named so? Each of them is obtained when a number is multiplied by taking it three times.

© NCERT not to be republished

We note that $1 = 1 \times 1 \times 1 = 1^3$; $8 = 2 \times 2 \times 2 = 2^3$; $27 = 3 \times 3 \times 3 = 3^3$.
Since $5^3 = 5 \times 5 \times 5 = 125$, therefore 125 is a cube number.

Is 9 a cube number? No, as $9 = 3 \times 3$ and there is no natural number which multiplied by taking three times gives 9. We can see also that $2 \times 2 \times 2 = 8$ and $3 \times 3 \times 3 = 27$. This shows that 9 is not a perfect cube.

The following are the cubes of numbers from 1 to 10.

Table 1

Number	Cube
1	$1^3 = 1$
2	$2^3 = 8$
3	$3^3 = 27$
4	$4^3 = 64$
5	$5^3 =$ ____
6	$6^3 =$ ____
7	$7^3 =$ ____
8	$8^3 =$ ____
9	$9^3 =$ ____
10	$10^3 =$ ____

The numbers 729, 1000, 1728 are also perfect cubes.

Complete it.

There are only ten perfect cubes from 1 to 1000. (Check this). How many perfect cubes are there from 1 to 100?

Observe the cubes of even numbers. Are they all even? What can you say about the cubes of odd numbers?

Following are the cubes of the numbers from 11 to 20.

Table 2

Number	Cube
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000

We are even, so are our cubes

We are odd so are our cubes

© NCERT not to be republished

Consider a few numbers having 1 as the one's digit (or unit's). Find the cube of each of them. What can you say about the one's digit of the cube of a number having 1 as the one's digit?
Similarly, explore the one's digit of cubes of numbers ending in 2, 3, 4, ... , etc.

TRY THESE

Find the one's digit of the cube of each of the following numbers.

(i) 3331 (ii) 8888 (iii) 149 (iv) 1005

(v) 1024 (vi) 77 (vii) 5022 (viii) 53

6.2.1 Some interesting patterns

1. Adding consecutive odd numbers

Observe the following pattern of sums of odd numbers.

$$1 = 1 = 1^3$$
$$3 + 5 = 8 = 2^3$$
$$7 + 9 + 11 = 27 = 3^3$$
$$13 + 15 + 17 + 19 = 64 = 4^3$$
$$21 + 23 + 25 + 27 + 29 = 125 = 5^3$$

Is it not interesting? How many consecutive odd numbers will be needed to obtain the sum as 10^3?

TRY THESE

Express the following numbers as the sum of odd numbers using the above pattern?

(a) 6^3 (b) 8^3 (c) 7^3

Consider the following pattern.

$$2^3 - 1^3 = 1 + 2 \times 1 \times 3$$
$$3^3 - 2^3 = 1 + 3 \times 2 \times 3$$
$$4^3 - 3^3 = 1 + 4 \times 3 \times 3$$

Using the above pattern, find the value of the following.

(i) $7^3 - 6^3$ (ii) $12^3 - 11^3$ (iii) $20^3 - 19^3$ (iv) $51^3 - 50^3$

2. Cubes and their prime factors

Consider the following prime factorisation of the numbers and their cubes.

Prime factorisation of a number	Prime factorisation of its cube
$4 = 2 \times 2$	$4^3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3$
$6 = 2 \times 3$	$6^3 = 216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3$
$15 = 3 \times 5$	$15^3 = 3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 = 3^3 \times 5^3$
$12 = 2 \times 2 \times 3$	$12^3 = 1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$ $= 2^3 \times 2^3 \times 3^3$

each prime factor appears three times in its cubes

© NCERT not to be republished

2	216
2	108
2	54
3	27
3	9
3	3
	1

Observe that each prime factor of a number appears three times in the prime factorisation of its cube.

Do you remember that $a^m \times b^m = (a \times b)^m$

In the prime factorisation of any number, if each factor appears three times, then, is the number a perfect cube? Think about it. Is 216 a perfect cube?

By prime factorisation, $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Each factor appears 3 times. $216 = 2^3 \times 3^3 = (2 \times 3)^3$

$= 6^3$ which is a perfect cube!

Is 729 a perfect cube? $729 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

factors can be grouped in triples

Yes, 729 is a perfect cube.

Now let us check for 500.

Prime factorisation of 500 is $2 \times 2 \times \underline{5 \times 5 \times 5}$.

So, 500 is not a perfect cube.

There are three 5's in the product but only two 2's.

Example 1: Is 243 a perfect cube?

Solution: $243 = \underline{3 \times 3 \times 3} \times 3 \times 3$

In the above factorisation 3×3 remains after grouping the 3's in triplets. Therefore, 243 is not a perfect cube.

TRY THESE

Which of the following are perfect cubes?

1. 400	2. 3375	3. 8000	4. 15625
5. 9000	6. 6859	7. 2025	8. 10648

6.2.2 Smallest multiple that is a perfect cube

Raj made a cuboid of plasticine. Length, breadth and height of the cuboid are 15 cm, 30 cm, 15 cm respectively.

Anu asks how many such cuboids will she need to make a perfect cube? Can you tell?

Raj said, Volume of cuboid is $15 \times 30 \times 15 = 3 \times 5 \times 2 \times 3 \times 5 \times 3 \times 5$

$= 2 \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

Since there is only one 2 in the prime factorisation. So we need 2×2, i.e., 4 to make it a perfect cube. Therefore, we need 4 such cuboids to make a cube.

Example 2: Is 392 a perfect cube? If not, find the smallest natural number by which 392 must be multiplied so that the product is a perfect cube.

Solution: $392 = \underline{2 \times 2 \times 2} \times 7 \times 7$

The prime factor 7 does not appear in a group of three. Therefore, 392 is not a perfect cube. To make its a cube, we need one more 7. In that case

$392 \times 7 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7} = 2744$ which is a perfect cube.

© NCERT not to be republished

Hence the smallest natural number by which 392 should be multiplied to make a perfect cube is 7.

Example 3: Is 53240 a perfect cube? If not, then by which smallest natural number should 53240 be divided so that the quotient is a perfect cube?

Solution: $53240 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11} \times 5$

The prime factor 5 does not appear in a group of three. So, 53240 is not a perfect cube. In the factorisation 5 appears only one time. If we divide the number by 5, then the prime factorisation of the quotient will not contain 5.

So, $53240 \div 5 = \underline{2 \times 2 \times 2} \times \underline{11 \times 11 \times 11}$

Hence the smallest number by which 53240 should be divided to make it a perfect cube is 5.

The perfect cube in that case is = 10648.

Example 4: Is 1188 a perfect cube? If not, by which smallest natural number should 1188 be divided so that the quotient is a perfect cube?

Solution: $1188 = 2 \times 2 \times \underline{3 \times 3 \times 3} \times 11$

The primes 2 and 11 do not appear in groups of three. So, 1188 is not a perfect cube. In the factorisation of 1188 the prime 2 appears only two times and the prime 11 appears once. So, if we divide 1188 by $2 \times 2 \times 11 = 44$, then the prime factorisation of the quotient will not contain 2 and 11.

Hence the smallest natural number by which 1188 should be divided to make it a perfect cube is 44.

And the resulting perfect cube is $1188 \div 44 = 27 (=3^3)$.

Example 5: Is 68600 a perfect cube? If not, find the smallest number by which 68600 must be multiplied to get a perfect cube.

Solution: We have, $68600 = 2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 7 \times 7$. In this factorisation, we find that there is no triplet of 5.

So, 68600 is not a perfect cube. To make it a perfect cube we multiply it by 5.

Thus, $68600 \times 5 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7$

$= 343000$, which is a perfect cube.

Observe that 343 is a perfect cube. From Example 5 we know that 343000 is also perfect cube.

THINK, DISCUSS AND WRITE

Check which of the following are perfect cubes. (i) 2700 (ii) 16000 (iii) 64000 (iv) 900 (v) 125000 (vi) 36000 (vii) 21600 (viii) 10,000 (ix) 27000000 (x) 1000. What pattern do you observe in these perfect cubes?

© NCERT not to be republished

EXERCISE 6.1

1. Which of the following numbers are not perfect cubes?
   (i) 216 (ii) 128 (iii) 1000 (iv) 100
   (v) 46656
2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.
   (i) 243 (ii) 256 (iii) 72 (iv) 675
   (v) 100
3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.
   (i) 81 (ii) 128 (iii) 135 (iv) 192
   (v) 704
4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

6.3 Cube Roots

If the volume of a cube is 125 cm^3, what would be the length of its side? To get the length of the side of the cube, we need to know a number whose cube is 125.

Finding the square root, as you know, is the inverse operation of squaring. Similarly, finding the cube root is the inverse operation of finding cube.

We know that $2^3 = 8$; so we say that the cube root of 8 is 2.

We write $\sqrt[3]{8} = 2$. **The symbol $\sqrt[3]{\ }$ denotes 'cube-root.'**

Consider the following:

Statement	Inference
$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = \sqrt[3]{2^3} = 2$
$3^3 = 27$	$\sqrt[3]{27} = \sqrt[3]{3^3} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$

Statement	Inference
$6^3 = 216$	$\sqrt[3]{216} = 6$
$7^3 = 343$	$\sqrt[3]{343} = 7$
$8^3 = 512$	$\sqrt[3]{512} = 8$
$9^3 = 729$	$\sqrt[3]{729} = 9$
$10^3 = 1000$	$\sqrt[3]{1000} = 10$

6.3.1 Cube root through prime factorisation method

Consider 3375. We find its cube root by prime factorisation:

$$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} = 3^3 \times 5^3 = (3 \times 5)^3$$

Therefore, cube root of 3375 = $\sqrt[3]{3375} = 3 \times 5 = 15$

Similarly, to find $\sqrt[3]{74088}$, we have,

© NCERT not to be republished

$$74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7} = 2^3 \times 3^3 \times 7^3 = (2 \times 3 \times 7)^3$$

Therefore, $\sqrt[3]{74088} = 2 \times 3 \times 7 = 42$

Example 6: Find the cube root of 8000.

Solution: Prime factorisation of 8000 is $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

So, $\sqrt[3]{8000} = 2 \times 2 \times 5 = 20$

Example 7: Find the cube root of 13824 by prime factorisation method.

Solution:

$$13824 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} = 2^3 \times 2^3 \times 2^3 \times 3^3.$$

Therefore, $\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$

THINK, DISCUSS AND WRITE

State true or false: for any integer m, $m^2 < m^3$. Why?

EXERCISE 6.2

1. Find the cube root of each of the following numbers by prime factorisation method.

(i) 64	(ii) 512	(iii) 10648	(iv) 27000
(v) 15625	(vi) 13824	(vii) 110592	(viii) 46656
(ix) 175616	(x) 91125		

2. State true or false.

(i) Cube of any odd number is even.
(ii) A perfect cube does not end with two zeros.
(iii) If square of a number ends with 5, then its cube ends with 25.
(iv) There is no perfect cube which ends with 8.
(v) The cube of a two digit number may be a three digit number.
(vi) The cube of a two digit number may have seven or more digits.
(vii) The cube of a single digit number may be a single digit number.

WHAT HAVE WE DISCUSSED?

1. Numbers like 1729, 4104, 13832, are known as Hardy – Ramanujan Numbers. They can be expressed as sum of two cubes in two different ways.

2. Numbers obtained when a number is multiplied by itself three times are known as **cube numbers**. For example 1, 8, 27, ... etc.

3. If in the prime factorisation of any number each factor appears three times, then the number is a perfect cube.

4. The symbol $\sqrt[3]{\ }$ denotes cube root. For example $\sqrt[3]{27} = 3$.

© NCERT not to be republished

NOTES

© NCERT not to be republished

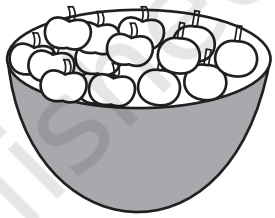
CHAPTER 7

Comparing Quantities

0852CH08

7.1 Recalling Ratios and Percentages

We know, ratio means comparing two quantities.
A basket has two types of fruits, say, 20 apples and 5 oranges.
Then, the ratio of the number of oranges to the number of apples = 5 : 20.



The comparison can be done by using fractions as, $\frac{5}{20} = \frac{1}{4}$

The number of oranges is $\frac{1}{4}$ th the number of apples. In terms of ratio, this is 1 : 4, read as, "1 is to 4"

OR

Number of apples to number of oranges = $\frac{20}{5} = \frac{4}{1}$ which means, the number of apples is 4 times the number of oranges. This comparison can also be done using percentages.

There are 5 oranges out of 25 fruits.
So percentage of oranges is

$$\frac{5}{25} \times \frac{4}{4} = \frac{20}{100} = 20\%$$

[Denominator made 100].

OR

By unitary method:
Out of 25 fruits, number of oranges are 5.
So out of 100 fruits, number of oranges

$$= \frac{5}{25} \times 100 = 20.$$

Since contains only apples and oranges,

So, percentage of apples + percentage of oranges = 100
or percentage of apples + 20 = 100
or percentage of apples = 100 – 20 = 80

Thus the basket has 20% oranges and 80% apples.

Example 1: A picnic is being planned in a school for Class VII. Girls are 60% of the total number of students and are 18 in number.

The picnic site is 55 km from the school and the transport company is charging at the rate of ₹ 12 per km. The total cost of refreshments will be ₹ 4280.

© NCERT not to be republished

Can you tell.

1. The ratio of the number of girls to the number of boys in the class?
2. The cost per head if two teachers are also going with the class?
3. If their first stop is at a place 22 km from the school, what per cent of the total distance of 55 km is this? What per cent of the distance is left to be covered?

Solution:

1. To find the ratio of girls to boys.

   Ashima and John came up with the following answers.

   They needed to know the number of boys and also the total number of students.

Ashima did this

Let the total number of students be x. 60% of x is girls.

Therefore, 60% of x = 18

$$\frac{60}{100} \times x = 18$$

or, $x = \frac{18 \times 100}{60} = 30$

Number of students = 30.

OR

John used the unitary method

There are 60 girls out of 100 students.

There is one girl out of $\frac{100}{60}$ students.

So, 18 girls are out of how many students?

Number of students = $\frac{100}{60} \times 18$

= 30

So, the number of boys = 30 – 18 = 12.

Hence, ratio of the number of girls to the number of boys is 18 : 12 or $\frac{18}{12} = \frac{3}{2}$.

$\frac{3}{2}$ is written as 3 : 2 and read as 3 is to 2.

2. To find the cost per person.

   Transportation charge = Distance both ways × Rate

   = ₹ (55 × 2) × 12

   = ₹ 110 × 12 = ₹ 1320

   Total expenses = Refreshment charge + Transportation charge

   = ₹ 4280 + ₹ 1320

   = ₹ 5600

   Total number of persons = 18 girls + 12 boys + 2 teachers

   = 32 persons

   Ashima and John then used unitary method to find the cost per head.

   For 32 persons, amount spent would be ₹ 5600.

   The amount spent for 1 person = ₹ $\frac{5600}{32}$ = ₹ 175.

3. The distance of the place where first stop was made = 22 km.

© NCERT not to be republished

To find the percentage of distance:

Ashima used this method:

$\frac{22}{55} = \frac{22}{55} \times \frac{100}{100} = 40\%$

She is multiplying the ratio by $\frac{100}{100} = 1$ and converting to percentage.

OR

John used the unitary method:

Out of 55 km, 22 km are travelled.

Out of 1 km, $\frac{22}{55}$ km are travelled.

Out of 100 km, $\frac{22}{55} \times 100$ km are travelled.

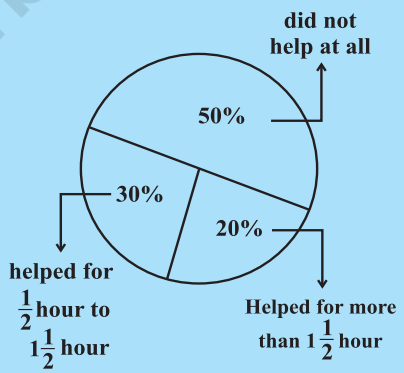
That is 40% of the total distance is travelled.

Both came out with the same answer that the distance from their school of the place where they stopped at was 40% of the total distance they had to travel.
Therefore, the percent distance left to be travelled = 100% – 40% = 60%.

TRY THESE

In a primary school, the parents were asked about the number of hours they spend per day in helping their children to do homework. There were 90 parents who helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$ hours. The distribution of parents according to the time for which, they said they helped is given in the adjoining figure ; 20% helped for more than $1\frac{1}{2}$ hours per day;
30% helped for $\frac{1}{2}$ hour to $1\frac{1}{2}$ hours; 50% did not help at all.
Using this, answer the following:

(i) How many parents were surveyed?
(ii) How many said that they did not help?
(iii) How many said that they helped for more than $1\frac{1}{2}$ hours?



EXERCISE 7.1

1. Find the ratio of the following.
   (a) Speed of a cycle 15 km per hour to the speed of scooter 30 km per hour.
   (b) 5 m to 10 km
   (c) 50 paise to ₹ 5
2. Convert the following ratios to percentages.
   (a) 3 : 4
   (b) 2 : 3
3. 72% of 25 students are interested in mathematics. How many are not interested in mathematics?
4. A football team won 10 matches out of the total number of matches they played. If their win percentage was 40, then how many matches did they play in all?
5. If Chameli had ₹ 600 left after spending 75% of her money, how much did she have in the beginning?

© NCERT not to be republished

6. If 60% people in a city like cricket, 30% like football and the remaining like other games, then what per cent of the people like other games? If the total number of people is 50 lakh, find the exact number who like each type of game.

7.2 Finding Discounts

Discount is a reduction given on the Marked Price (MP) of the article.

This is generally given to attract customers to buy goods or to promote sales of the goods. You can find the discount by subtracting its sale price from its marked price.

So, Discount = Marked price – Sale price

Example 2: An item marked at ₹ 840 is sold for ₹ 714. What is the discount and discount %?

Solution:

Discount = Marked Price – Sale Price
= ₹ 840 – ₹ 714
= ₹ 126

Since discount is on marked price, we will have to use marked price as the base.

On marked price of ₹ 840, the discount is ₹ 126.
On MP of ₹ 100, how much will the discount be?

$$\text{Discount} = \frac{126}{840} \times 100\% = 15\%$$

You can also find discount when discount % is given.

Example 3: The list price of a frock is ₹ 220. A discount of 20% is announced on sales. What is the amount of discount on it and its sale price.

Solution: Marked price is same as the list price.
20% discount means that on ₹ 100 (MP), the discount is ₹ 20.

By unitary method, on ₹1 the discount will be ₹ $\frac{20}{100}$.

On ₹ 220, discount = ₹ $\frac{20}{100} \times 220$ = ₹ 44

The sale price = (₹ 220 – ₹ 44) or ₹ 176

Rehana found the sale price like this —

A discount of 20% means for a MP of ₹ 100, discount is ₹ 20. Hence the sale price is ₹ 80. Using unitary method, when MP is ₹ 100, sale price is ₹ 80;

© NCERT not to be republished

When MP is ₹ 1, sale price is ₹ $\frac{80}{100}$.

Hence when MP is ₹ 220, sale price = ₹ $\frac{80}{100} \times 220$ = ₹ 176.

Even though the discount was not found, I could find the sale price directly.

TRY THESE

1. A shop gives 20% discount. What would the sale price of each of these be?
   (a) A dress marked at ₹ 120 (b) A pair of shoes marked at ₹ 750
   (c) A bag marked at ₹ 250
2. A table marked at ₹ 15,000 is available for ₹ 14,400. Find the discount given and the discount per cent.
3. An almirah is sold at ₹ 5,225 after allowing a discount of 5%. Find its marked price.

7.2.1 Estimation in percentages

Your bill in a shop is ₹ 577.80 and the shopkeeper gives a discount of 15%. How would you estimate the amount to be paid?

(i) Round off the bill to the nearest tens of ₹ 577.80, i.e., to ₹ 580.

(ii) Find 10% of this, i.e., ₹ $\frac{10}{100} \times 580$ = ₹ 58 .

(iii) Take half of this, i.e., $\frac{1}{2} \times 58$ = ₹ 29 .

(iv) Add the amounts in (ii) and (iii) to get ₹ 87.

You could therefore reduce your bill amount by ₹ 87 or by about ₹ 85, which will be ₹ 495 approximately.

1. Try estimating 20% of the same bill amount. 2. Try finding 15% of ₹ 375.

7.3 Sales Tax/Value Added Tax/Goods and Services Tax

The teacher showed the class a bill in which the following heads were written.

Bill No.			Date	
Menu				
S.No.	Item	Quantity	Rate	Amount
		Bill amount + ST (5%)		
	Total			

© NCERT not to be republished

Sales tax (ST) is charged by the government on the sale of an item. It is collected by the shopkeeper from the customer and given to the government. This is, therefore, always on the selling price of an item and is added to the value of the bill. There is another type of tax which is included in the prices known as **Value Added Tax (VAT)**.

From July 1, 2017, Government of India introduced GST which stands for Goods and Services Tax which is levied on supply of goods or services or both.

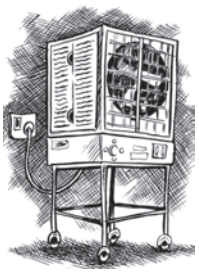
Example 4: (**Finding Sales Tax**) The cost of a pair of roller skates at a shop was ₹ 450. The sales tax charged was 5%. Find the bill amount.

Solution: On ₹ 100, the tax paid was ₹ 5.

$$\text{On ₹ 450, the tax paid would be} = ₹ \frac{5}{100} \times 450$$

$$= ₹ 22.50$$

Bill amount = Cost of item + Sales tax = ₹ 450 + ₹ 22.50 = ₹ 472.50.



Example 5: (**Value Added Tax (VAT)**) Waheeda bought an air cooler for ₹ 3300 including a tax of 10%. Find the price of the air cooler before VAT was added.

Solution: The price includes the VAT, i.e., the value added tax. Thus, a 10% VAT means if the price without VAT is ₹ 100 then price including VAT is ₹ 110.

Now, when price including VAT is ₹ 110, original price is ₹ 100.

Hence when price including tax is ₹ 3300, the original price = $₹ \frac{100}{110} \times 3300 = ₹ 3000$.

Example 6: Salim bought an article for ₹ 784 which included GST of 12% . What is the price of the article before GST was added?

Solution: Let original price of the article be ₹ 100. GST = 12%.

Price after GST is included = ₹ (100+12) = ₹ 112

When the selling price is ₹ 112 then original price = ₹ 100.

When the selling price is ₹ 784, then original price = $₹ \frac{100}{12} \times 784 = ₹ 700$

THINK, DISCUSS AND WRITE

1. Two times a number is a 100% increase in the number. If we take half the number what would be the decrease in per cent?
2. By what per cent is ₹ 2,000 less than ₹ 2,400? Is it the same as the per cent by which ₹ 2,400 is more than ₹ 2,000?

© NCERT not to be republished

EXERCISE 7.2

1. During a sale, a shop offered a discount of 10% on the marked prices of all the items. What would a customer have to pay for a pair of jeans marked at ₹ 1450 and two shirts marked at ₹ 850 each?
2. The price of a TV is ₹ 13,000. The sales tax charged on it is at the rate of 12%. Find the amount that Vinod will have to pay if he buys it.
3. Arun bought a pair of skates at a sale where the discount given was 20%. If the amount he pays is ₹ 1,600, find the marked price.
4. I purchased a hair-dryer for ₹ 5,400 including 8% VAT. Find the price before VAT was added.
5. An article was purchased for ₹ 1239 including GST of 18%. Find the price of the article before GST was added?

7.4 Compound Interest

You might have come across statements like "one year interest for FD (fixed deposit) in the bank @ 9% per annum" or 'Savings account with interest @ 5% per annum'.

Interest is the extra money paid by institutions like banks or post offices on money deposited (kept) with them. Interest is also paid by people when they borrow money. We already know how to calculate **Simple Interest**.

Example 7: A sum of ₹ 10,000 is borrowed at a rate of interest 15% per annum for 2 years. Find the simple interest on this sum and the amount to be paid at the end of 2 years.

Solution: On ₹ 100, interest charged for 1 year is ₹ 15.

So, on ₹ 10,000, interest charged $= \frac{15}{100} \times 10000 =$ ₹ 1500

Interest for 2 years = ₹ 1500 × 2 = ₹ 3000

Amount to be paid at the end of 2 years = Principal + Interest

= ₹ 10000 + ₹ 3000 = ₹ 13000

TRY THESE

Find interest and amount to be paid on ₹ 15000 at 5% per annum after 2 years.

My father has kept some money in the post office for 3 years. Every year the money increases as more than the previous year.

We have some money in the bank. Every year some interest is added to it, which is shown in the passbook. This interest is not the same, each year it increases.

© NCERT not to be republished

Normally, the interest paid or charged is never simple. The interest is calculated on the amount of the previous year. This is known as interest compounded or **Compound Interest** (C.I.).

Let us take an example and find the interest year by year. Each year our sum or principal changes.

Calculating Compound Interest

A sum of ₹ 20,000 is borrowed by Heena for 2 years at an interest of 8% compounded annually. Find the Compound Interest (C.I.) and the amount she has to pay at the end of 2 years.

Aslam asked the teacher whether this means that they should find the interest year by year. The teacher said 'yes', and asked him to use the following steps :

1. Find the Simple Interest (S.I.) for one year.
   Let the principal for the first year be P_1. Here, P_1 = ₹ 20,000

$$SI_1 = \text{SI at } 8\% \text{ p.a. for 1st year} = ₹ \frac{20000 \times 8}{100} = ₹ 1600$$

2. Then find the amount which will be paid or received. This becomes principal for the next year.

$$\text{Amount at the end of 1st year} = P_1 + SI_1 = ₹ 20000 + ₹ 1600$$
$$= ₹ 21600 = P_2 \text{ (Principal for 2nd year)}$$

3. Again find the interest on this sum for another year.

$$SI_2 = \text{SI at } 8\% \text{ p.a.for 2nd year} = ₹ \frac{21600 \times 8}{100}$$
$$= ₹ 1728$$

4. Find the amount which has to be paid or received at the end of second year.

$$\text{Amount at the end of 2nd year} = P_2 + SI_2$$
$$= ₹ 21600 + ₹ 1728$$
$$= ₹ 23328$$
$$\text{Total interest given} = ₹ 1600 + ₹ 1728$$
$$= ₹ 3328$$

Reeta asked whether the amount would be different for simple interest. The teacher told her to find the interest for two years and see for herself.

$$\text{SI for 2 years} = ₹ \frac{20000 \times 8 \times 2}{100} = ₹ 3200$$

Reeta said that when compound interest was used Heena would pay ₹ 128 more.

Let us look at the difference between simple interest and compound interest. We start with ₹ 100. Try completing the chart.

© NCERT not to be republished

		Under Simple Interest	Under Compound Interest
First year	Principal	₹ 100.00	₹ 100.00
	Interest at 10%	₹ 10.00	₹ 10.00
	Year-end amount	₹ 110.00	₹ 110.00
Second year	Principal	₹ 100.00	₹ 110.00
	Interest at 10%	₹ 10.00	₹ 11.00
	Year-end amount	₹(110 + 10) = ₹ 120	₹ 121.00
Third year	Principal	₹ 100.00	₹ 121.00
	Interest at 10%	₹ 10.00	₹ 12.10
	Year-end amount	₹(120 + 10) = ₹ 130	₹ 133.10

Which means you pay interest on the interest accumulated till then!

Note that in 3 years,

Interest earned by Simple Interest = ₹ (130 – 100) = ₹ 30, whereas,

Interest earned by Compound Interest = ₹ (133.10 – 100) = ₹ 33.10

Note also that the Principal remains the same under Simple Interest, while it changes year after year under compound interest.

7.5 Deducing a Formula for Compound Interest

Zubeda asked her teacher, 'Is there an easier way to find compound interest?' The teacher said 'There is a shorter way of finding compound interest. Let us try to find it.'

Suppose P_1 is the sum on which interest is compounded annually at a rate of R% per annum.

Let P_1 = ₹ 5000 and R = 5. Then by the steps mentioned above

1. $SI_1 = ₹ \frac{5000 \times 5 \times 1}{100}$ or $SI_1 = ₹ \frac{P_1 \times R \times 1}{100}$

so, $A_1 = ₹ 5000 + \frac{5000 \times 5 \times 1}{100}$ or $A_1 = P_1 + SI_1 = P_1 + \frac{P_1 R}{100}$

$= ₹ 5000 \left(1 + \frac{5}{100}\right) = P_2$ $= P_1 \left(1 + \frac{R}{100}\right) = P_2$

© NCERT not to be republished

2. $SI_2 = ₹\ 5000\left(1+\frac{5}{100}\right)\times\frac{5\times1}{100}$ or $SI_2 = \frac{P_2\times R\times1}{100}$

$= ₹\ \frac{5000\times5}{100}\left(1+\frac{5}{100}\right)$ $= P_1\left(1+\frac{R}{100}\right)\times\frac{R}{100}$

$= \frac{P_1R}{100}\left(1+\frac{R}{100}\right)$

$A_2 = ₹\ 5000\left(1+\frac{5}{100}\right)+ ₹\ \frac{5000\times5}{100}\left(1+\frac{5}{100}\right)$ $A_2 = P_2 + SI_2$

$= ₹\ 5000\left(1+\frac{5}{100}\right)\left(1+\frac{5}{100}\right)$ $= P_1\left(1+\frac{R}{100}\right)+P_1\frac{R}{100}\left(1+\frac{R}{100}\right)$

$= ₹\ 5000\left(1+\frac{5}{100}\right)^2 = P_3$ $= P_1\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right)$

$= P_1\left(1+\frac{R}{100}\right)^2 = P_3$

Proceeding in this way the amount at the end of *n* years will be

$$A_n = P_1\left(1+\frac{R}{100}\right)^n$$

Or, we can say $A = P\left(1+\frac{R}{100}\right)^n$

So, Zubeda said, but using this we get only the formula for the amount to be paid at the end of *n* years, and not the formula for compound interest.

Aruna at once said that we know CI = A – P, so we can easily find the compound interest too.

Example 8: Find CI on ₹ 12600 for 2 years at 10% per annum compounded annually.

Solution: We have, $A = P\left(1+\frac{R}{100}\right)^n$, where Principal (P) = ₹ 12600, Rate (R) = 10, Number of years (*n*) = 2

$$= ₹\ 12600\left(1+\frac{10}{100}\right)^2 = ₹\ 12600\left(\frac{11}{10}\right)^2$$

© NCERT not to be republished

$$= ₹\ 12600 \times \frac{11}{10} \times \frac{11}{10} = ₹\ 15246$$

$$CI = A - P = ₹\ 15246 - ₹\ 12600 = ₹\ 2646$$

TRY THESE

1. Find CI on a sum of ₹ 8000 for 2 years at 5% per annum compounded annually.

7.6 Applications of Compound Interest Formula

There are some situations where we could use the formula for calculation of amount in CI. Here are a few.

(i) Increase (or decrease) in population.

(ii) The growth of a bacteria if the rate of growth is known.

(iii) The value of an item, if its price increases or decreases in the intermediate years.

Example 9: The population of a city was 20,000 in the year 1997. It increased at the rate of 5% p.a. Find the population at the end of the year 2000.

Solution: There is 5% increase in population every year, so every new year has new population. Thus, we can say it is increasing in compounded form.

Population in the beginning of 1998 = 20000 (we treat this as the principal for the 1st year)

$$\text{Increase at } 5\% = \frac{5}{100} \times 20000 = 1000$$

$$\text{Population in } 1999 = 20000 + 1000 = 21000$$

Treat as the Principal for the 2nd year.

$$\text{Increase at } 5\% = \frac{5}{100} \times 21000 = 1050$$

$$\text{Population in } 2000 = 21000 + 1050$$

$$= 22050$$

Treat as the Principal for the 3rd year.

$$\text{Increase at } 5\% = \frac{5}{100} \times 22050$$

$$= 1102.5$$

$$\text{At the end of 2000 the population} = 22050 + 1102.5 = 23152.5$$

or, $$\text{Population at the end of 2000} = 20000 \left(1 + \frac{5}{100}\right)^3$$

$$= 20000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$= 23152.5$$

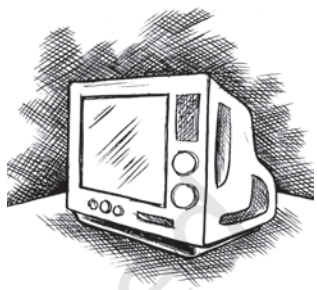
So, the estimated population = 23153.

© NCERT not to be republished

Aruna asked what is to be done if there is a decrease. The teacher then considered the following example.

Example 10: A TV was bought at a price of ₹ 21,000. After one year the value of the TV was depreciated by 5% (Depreciation means reduction of value due to use and age of the item). Find the value of the TV after one year.

Solution:



Principal = ₹ 21,000

Reduction = 5% of ₹ 21000 per year

$$= ₹ \frac{21000 \times 5 \times 1}{100} = ₹ 1050$$

value at the end of 1 year = ₹ 21000 – ₹ 1050 = ₹ 19,950

Alternately, We may directly get this as follows:

$$\text{value at the end of 1 year} = ₹ 21000 \left(1 - \frac{5}{100}\right)$$

$$= ₹ 21000 \times \frac{19}{20} = ₹ 19,950$$

TRY THESE

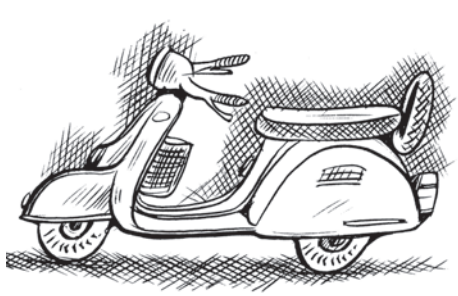
1. A machinery worth ₹ 10,500 depreciated by 5%. Find its value after one year.
2. Find the population of a city after 2 years, which is at present 12 lakh, if the rate of increase is 4%.

EXERCISE 7.3

1. The population of a place increased to 54,000 in 2003 at a rate of 5% per annum
   (i) find the population in 2001.
   (ii) what would be its population in 2005?

2. In a Laboratory, the count of bacteria in a certain experiment was increasing at the rate of 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially 5, 06,000.

3. A scooter was bought at ₹ 42,000. Its value depreciated at the rate of 8% per annum. Find its value after one year.



© NCERT not to be republished

WHAT HAVE WE DISCUSSED?

1. **Discount** is a reduction given on marked price.
   Discount = Marked Price – Sale Price.
2. Discount can be calculated when discount percentage is given.
   Discount = Discount % of Marked Price
3. Additional expenses made after buying an article are included in the cost price and are known as **overhead expenses**.
   CP = Buying price + Overhead expenses
4. Sales tax is charged on the sale of an item by the government and is added to the Bill Amount.
   Sales tax = Tax% of Bill Amount
5. GST stands for Goods and Services Tax and is levied on supply of goods or services or both.
6. Compound interest is the interest calculated on the previous year's amount (A = P + I)

© NCERT not to be republished

NOTES

© NCERT not to be republished

CHAPTER

8

Algebraic Expressions and Identities

0852CH09

8.1 Addition and Subtraction of Algebraic Expressions

In earlier classes, we have already become familiar with what algebraic expressions (or simply expressions) are. Examples of expressions are:

$$x + 3,\ 2y - 5,\ 3x^2,\ 4xy + 7 \text{ etc.}$$

In the earlier classes, we have also learnt how to add and subtract algebraic expressions. For example, to add $7x^2 - 4x + 5$ and $9x - 10$, we do

$$\begin{array}{rr} & 7x^2 - 4x + 5 \\ + & \quad 9x - 10 \\ \hline & 7x^2 + 5x - 5 \end{array}$$

Observe how we do the addition. We write each expression to be added in a separate row. While doing so we write like terms one below the other, and add them, as shown. Thus $5 + (-10) = 5 - 10 = -5$. Similarly, $-4x + 9x = (-4 + 9)x = 5x$. Let us take some more examples.

Example 1: Add: $7xy + 5yz - 3zx$, $4yz + 9zx - 4y$, $-3xz + 5x - 2xy$.

Solution: Writing the three expressions in separate rows, with like terms one below the other, we have

$$\begin{array}{rrrrr} & 7xy & + 5yz & -3zx & \\ + & & 4yz + & 9zx & - 4y \\ + & -2xy & & - 3zx + 5x & \\ \hline & 5xy & + 9yz + & 3zx + 5x & - 4y \end{array}$$

(Note xz is same as zx)

Thus, the sum of the expressions is $5xy + 9yz + 3zx + 5x - 4y$. Note how the terms, $-4y$ in the second expression and $5x$ in the third expression, are carried over as they are, since they have no like terms in the other expressions.

© NCERT not to be republished

Example 2: Subtract $5x^2 - 4y^2 + 6y - 3$ from $7x^2 - 4xy + 8y^2 + 5x - 3y$.

Solution:

$$\begin{array}{llll} 7x^2 - 4xy + 8y^2 + 5x - 3y & & & \\ 5x^2 & - 4y^2 & + 6y - 3 & \\ \hline (-) & (+) & (-) & (+) \end{array}$$

$$2x^2 - 4xy + 12y^2 + 5x - 9y + 3$$

Note that subtraction of a number is the same as addition of its additive inverse. Thus subtracting –3 is the same as adding +3. Similarly, subtracting $6y$ is the same as adding $-6y$; subtracting $-4y^2$ is the same as adding $4y^2$ and so on. The signs in the third row written below each term in the second row help us in knowing which operation has to be performed.

EXERCISE 8.1

1. Add the following.

(i) $ab - bc, bc - ca, ca - ab$

(ii) $a - b + ab, b - c + bc, c - a + ac$

(iii) $2p^2q^2 - 3pq + 4, 5 + 7pq - 3p^2q^2$

(iv) $l^2 + m^2, m^2 + n^2, n^2 + l^2, 2lm + 2mn + 2nl$

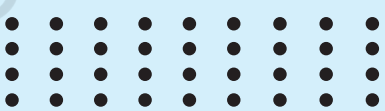
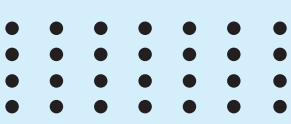
2. (a) Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$

(b) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

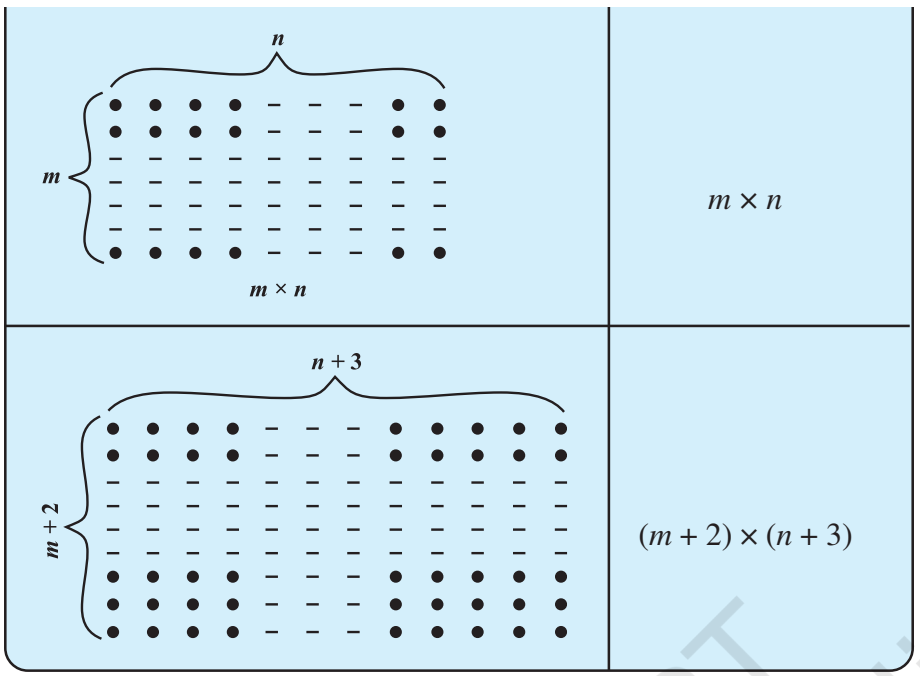
(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$

8.2 Multiplication of Algebraic Expressions: Introduction

(i) Look at the following patterns of dots.

Pattern of dots	Total number of dots
	4×9
	5×7

© NCERT not to be republished

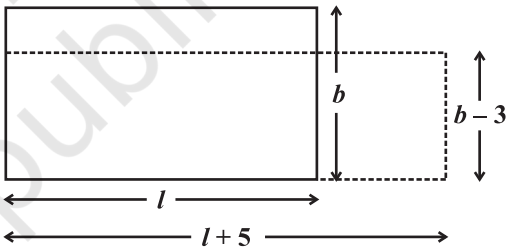


To find the number of dots we have to multiply the expression for the number of rows by the expression for the number of columns.

Here the number of rows is increased by 2, i.e., $m + 2$ and number of columns increased by 3, i.e., $n + 3$.

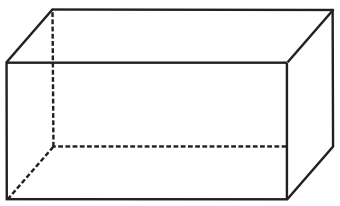
(ii) Can you now think of similar other situations in which two algebraic expressions have to be multiplied?

Ameena gets up. She says, "We can think of area of a rectangle." The area of a rectangle is $l \times b$, where l is the length, and b is breadth. If the length of the rectangle is increased by 5 units, i.e., $(l + 5)$ and breadth is decreased by 3 units , i.e., $(b - 3)$ units, the area of the new rectangle will be $(l + 5) \times (b - 3)$.



To find the area of a rectangle, we have to multiply algebraic expressions like $l \times b$ or $(l + 5) \times (b - 3)$.

(iii) Can you think about volume? (The volume of a rectangular box is given by the product of its length, breadth and height).



(iv) Sarita points out that when we buy things, we have to carry out multiplication. For example, if

price of bananas per dozen = ₹ p

and for the school picnic bananas needed = z dozens,

then we have to pay = ₹ $p \times z$

Suppose, the price per dozen was less by ₹ 2 and the bananas needed were less by 4 dozens.

Then, price of bananas per dozen = ₹ $(p - 2)$

and bananas needed = $(z - 4)$ dozens,

Therefore, we would have to pay = ₹ $(p - 2) \times (z - 4)$

© NCERT not to be republished

TRY THESE

Can you think of two more such situations, where we may need to multiply algebraic expressions?

[**Hint:** • Think of speed and time;
• Think of interest to be paid, the principal and the rate of simple interest; etc.]

In all the above examples, we had to carry out multiplication of two or more quantities. If the quantities are given by algebraic expressions, we need to find their product. This means that we should know how to obtain this product. Let us do this systematically. To begin with we shall look at the multiplication of two monomials.

8.3 Multiplying a Monomial by a Monomial

Expression that contains only one term is called a **monomial**.

8.3.1 Multiplying two monomials

We begin with

$4 \times x = x + x + x + x = 4x$ as seen earlier.

Similarly, $4 \times (3x) = 3x + 3x + 3x + 3x = 12x$

Now, observe the following products.

(i) $x \times 3y = x \times 3 \times y = 3 \times x \times y = 3xy$

(ii) $5x \times 3y = 5 \times x \times 3 \times y = 5 \times 3 \times x \times y = 15xy$

(iii) $5x \times (-3y) = 5 \times x \times (-3) \times y$
$= 5 \times (-3) \times x \times y = -15xy$

Notice that all the three products of monomials, $3xy$, $15xy$, $-15xy$, are also monomials.

Some more useful examples follow.

(iv) $5x \times 4x^2 = (5 \times 4) \times (x \times x^2)$
$= 20 \times x^3 = 20x^3$

(v) $5x \times (-4xyz) = (5 \times -4) \times (x \times xyz)$
$= -20 \times (x \times x \times yz) = -20x^2yz$

Observe how we collect the powers of different variables in the algebraic parts of the two monomials. While doing so, we use the rules of exponents and powers.

Note that $5 \times 4 = 20$
i.e., coefficient of product = coefficient of first monomial × coefficient of second monomial;
and $x \times x^2 = x^3$
i.e., algebraic factor of product = algebraic factor of first monomial × algebraic factor of second monomial.

8.3.2 Multiplying three or more monomials

Observe the following examples.

(i) $2x \times 5y \times 7z = (2x \times 5y) \times 7z = 10xy \times 7z = 70xyz$

(ii) $4xy \times 5x^2y^2 \times 6x^3y^3 = (4xy \times 5x^2y^2) \times 6x^3y^3 = 20x^3y^3 \times 6x^3y^3 = 120x^3y^3 \times x^3y^3$
$= 120\,(x^3 \times x^3) \times (y^3 \times y^3) = 120x^6 \times y^6 = 120x^6y^6$

It is clear that we first multiply the first two monomials and then multiply the resulting monomial by the third monomial. This method can be extended to the product of any number of monomials.

© NCERT not to be republished

TRY THESE

Find $4x \times 5y \times 7z$
First find $4x \times 5y$ and multiply it by $7z$;
or first find $5y \times 7z$ and multiply it by $4x$.
Is the result the same? What do you observe?
Does the order in which you carry out the multiplication matter?

We can find the product in other way also.
$4xy \times 5x^2y^2 \times 6x^3 y^3$
$= (4 \times 5 \times 6) \times (x \times x^2 \times x^3) \times (y \times y^2 \times y^3)$
$= 120\ x^6y^6$

Example 3: Complete the table for area of a rectangle with given length and breadth.

Solution:

length	breadth	area
$3x$	$5y$	$3x \times 5y = 15xy$
$9y$	$4y^2$	.....
$4ab$	$5bc$	.....
$2l^2m$	$3lm^2$	.....

Example 4: Find the volume of each rectangular box with given length, breadth and height.

	length	breadth	height
(i)	$2ax$	$3by$	$5cz$
(ii)	m^2n	n^2p	p^2m
(iii)	$2q$	$4q^2$	$8q^3$

Solution: Volume = length × breadth × height

Hence, for (i) volume $= (2ax) \times (3by) \times (5cz)$
$= 2 \times 3 \times 5 \times (ax) \times (by) \times (cz) = 30abcxyz$

for (ii) volume $= m^2n \times n^2p \times p^2m$
$= (m^2 \times m) \times (n \times n^2) \times (p \times p^2) = m^3n^3p^3$

for (iii) volume $= 2q \times 4q^2 \times 8q^3$
$= 2 \times 4 \times 8 \times q \times q^2 \times q^3 = 64q^6$

EXERCISE 8.2

1. Find the product of the following pairs of monomials.

(i) $4, 7p$ (ii) $-4p, 7p$ (iii) $-4p, 7pq$ (iv) $4p^3, -3p$
(v) $4p, 0$

2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively.

(p, q); $(10m, 5n)$; $(20x^2, 5y^2)$; $(4x, 3x^2)$; $(3mn, 4np)$

© NCERT not to be republished

3. Complete the table of products.

First monomial → / Second monomial ↓	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	...	...	...	...	...
$-5y$	...	...	$-15x^2y$	...	...	...
$3x^2$	...	...	...	...	...	...
$-4xy$	...	...	...	...	...	...
$7x^2y$	...	...	...	...	...	...
$-9x^2y^2$	...	...	...	...	...	...

4. Obtain the volume of rectangular boxes with the following length, breadth and height respectively.

(i) $5a, 3a^2, 7a^4$ (ii) $2p, 4q, 8r$ (iii) $xy, 2x^2y, 2xy^2$ (iv) $a, 2b, 3c$

5. Obtain the product of

(i) xy, yz, zx (ii) $a, -a^2, a^3$ (iii) $2, 4y, 8y^2, 16y^3$

(iv) $a, 2b, 3c, 6abc$ (v) $m, -mn, mnp$

8.4 Multiplying a Monomial by a Polynomial

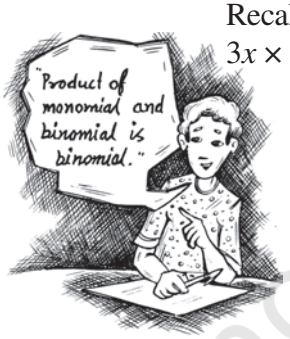
Expression that contains two terms is called a **binomial**. An expression containing three terms is a **trinomial** and so on. In general, an expression containing, one or more terms with non-zero coefficient (with variables having non negative integers as exponents) is called a **polynomial**.

8.4.1 Multiplying a monomial by a binomial

Let us multiply the monomial $3x$ by the binomial $5y + 2$, i.e., find $3x \times (5y + 2) = ?$

Recall that $3x$ and $(5y + 2)$ represent numbers. Therefore, using the distributive law,

$3x \times (5y + 2) = (3x \times 5y) + (3x \times 2) = 15xy + 6x$



We commonly use distributive law in our calculations. For example:

$7 \times 106 = 7 \times (100 + 6)$

$= 7 \times 100 + 7 \times 6$ (Here, we used distributive law)

$= 700 + 42 = 742$

$7 \times 38 = 7 \times (40 - 2)$

$= 7 \times 40 - 7 \times 2$ (Here, we used distributive law)

$= 280 - 14 = 266$

Similarly, $(-3x) \times (-5y + 2) = (-3x) \times (-5y) + (-3x) \times (2) = 15xy - 6x$

and $5xy \times (y^2 + 3) = (5xy \times y^2) + (5xy \times 3) = 5xy^3 + 15xy$.

What about a binomial × monomial? For example, $(5y + 2) \times 3x = ?$

We may use commutative law as : $7 \times 3 = 3 \times 7$; or in general $a \times b = b \times a$

Similarly, $(5y + 2) \times 3x = 3x \times (5y + 2) = 15xy + 6x$ as before.

TRY THESE

Find the product (i) $2x\ (3x + 5xy)$ (ii) $a^2\ (2ab - 5c)$

© NCERT not to be republished

8.4.2 Multiplying a monomial by a trinomial

Consider $3p \times (4p^2 + 5p + 7)$. As in the earlier case, we use distributive law;

$$3p \times (4p^2 + 5p + 7) = (3p \times 4p^2) + (3p \times 5p) + (3p \times 7)$$
$$= 12p^3 + 15p^2 + 21p$$

Multiply each term of the trinomial by the monomial and add products.

Observe, by using the distributive law, we are able to carry out the multiplication term by term.

TRY THESE

Find the product:
$(4p^2 + 5p + 7) \times 3p$

Example 5: Simplify the expressions and evaluate them as directed:

(i) $x\,(x - 3) + 2$ for $x = 1$, (ii) $3y\,(2y - 7) - 3\,(y - 4) - 63$ for $y = -2$

Solution:

(i) $x\,(x - 3) + 2 = x^2 - 3x + 2$

For $x = 1$, $x^2 - 3x + 2 = (1)^2 - 3\,(1) + 2$
$= 1 - 3 + 2 = 3 - 3 = 0$

(ii) $3y\,(2y - 7) - 3\,(y - 4) - 63 = 6y^2 - 21y - 3y + 12 - 63$
$= 6y^2 - 24y - 51$

For $y = -2$, $6y^2 - 24y - 51 = 6\,(-2)^2 - 24(-2) - 51$
$= 6 \times 4 + 24 \times 2 - 51$
$= 24 + 48 - 51 = 72 - 51 = 21$

Example 6: Add

(i) $5m\,(3 - m)$ and $6m^2 - 13m$ (ii) $4y\,(3y^2 + 5y - 7)$ and $2\,(y^3 - 4y^2 + 5)$

Solution:

(i) First expression $= 5m\,(3 - m) = (5m \times 3) - (5m \times m) = 15m - 5m^2$

Now adding the second expression to it, $15m - 5m^2 + 6m^2 - 13m = m^2 + 2m$

(ii) The first expression $= 4y\,(3y^2 + 5y - 7) = (4y \times 3y^2) + (4y \times 5y) + (4y \times (-7))$
$= 12y^3 + 20y^2 - 28y$

The second expression $= 2\,(y^3 - 4y^2 + 5) = 2y^3 + 2 \times (-4y^2) + 2 \times 5$
$= 2y^3 - 8y^2 + 10$

Adding the two expressions,

$$\begin{array}{rrrrr} 12y^3 & + & 20y^2 - 28y & & \\ + \quad 2y^3 & - & 8y^2 & + 10 & \\ \hline 14y^3 & + & 12y^2 - 28y & + 10 & \end{array}$$

Example 7: Subtract $3pq\,(p - q)$ from $2pq\,(p + q)$.

Solution: We have $3pq\,(p - q) = 3p^2q - 3pq^2$ and

$2pq\,(p + q) = 2p^2q + 2pq^2$

Subtracting,

$$\begin{array}{rrr} 2p^2q & + & 2pq^2 \\ 3p^2q & - & 3pq^2 \\ \hline -p^2q & + & 5pq^2 \end{array}$$

© NCERT not to be republished

EXERCISE 8.3

1. Carry out the multiplication of the expressions in each of the following pairs.

(i) $4p, q + r$ (ii) $ab, a - b$ (iii) $a + b, 7a^2b^2$ (iv) $a^2 - 9, 4a$

(v) $pq + qr + rp, 0$

2. Complete the table.

	First expression	Second expression	Product
(i)	a	$b + c + d$	...
(ii)	$x + y - 5$	$5xy$	...
(iii)	p	$6p^2 - 7p + 5$	...
(iv)	$4p^2q^2$	$p^2 - q^2$	...
(v)	$a + b + c$	abc	...

3. Find the product.

(i) $(a^2) \times (2a^{22}) \times (4a^{26})$ (ii) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$

(iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$ (iv) $x \times x^2 \times x^3 \times x^4$

4. (a) Simplify $3x(4x - 5) + 3$ and find its values for (i) $x = 3$ (ii) $x = \frac{1}{2}$.

(b) Simplify $a(a^2 + a + 1) + 5$ and find its value for (i) $a = 0$, (ii) $a = 1$ (iii) $a = -1$.

5. (a) Add: $p(p - q), q(q - r)$ and $r(r - p)$

(b) Add: $2x(z - x - y)$ and $2y(z - y - x)$

(c) Subtract: $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$

(d) Subtract: $3a(a + b + c) - 2b(a - b + c)$ from $4c(-a + b + c)$

8.5 Multiplying a Polynomial by a Polynomial

8.5.1 Multiplying a binomial by a binomial

Let us multiply one binomial $(2a + 3b)$ by another binomial, say $(3a + 4b)$. We do this step-by-step, as we did in earlier cases, following the distributive law of multiplication,

$$(3a + 4b) \times (2a + 3b) = 3a \times (2a + 3b) + 4b \times (2a + 3b)$$

Observe, every term in one binomial multiplies every term in the other binomial.

$$= (3a \times 2a) + (3a \times 3b) + (4b \times 2a) + (4b \times 3b)$$
$$= 6a^2 + 9ab + 8ba + 12b^2$$
$$= 6a^2 + 17ab + 12b^2 \quad (\text{Since } ba = ab)$$

When we carry out term by term multiplication, we expect $2 \times 2 = 4$ terms to be present. But two of these are like terms, which are combined, and hence we get 3 terms. **In multiplication of polynomials with polynomials, we should always look for like terms, if any, and combine them.**

© NCERT not to be republished

Example 8: Multiply

(i) $(x - 4)$ and $(2x + 3)$ (ii) $(x - y)$ and $(3x + 5y)$

Solution:

(i) $(x - 4) \times (2x + 3) = x \times (2x + 3) - 4 \times (2x + 3)$
$= (x \times 2x) + (x \times 3) - (4 \times 2x) - (4 \times 3) = 2x^2 + 3x - 8x - 12$
$= 2x^2 - 5x - 12$ (Adding like terms)

(ii) $(x - y) \times (3x + 5y) = x \times (3x + 5y) - y \times (3x + 5y)$
$= (x \times 3x) + (x \times 5y) - (y \times 3x) - (y \times 5y)$
$= 3x^2 + 5xy - 3yx - 5y^2 = 3x^2 + 2xy - 5y^2$ (Adding like terms)

Example 9: Multiply

(i) $(a + 7)$ and $(b - 5)$ (ii) $(a^2 + 2b^2)$ and $(5a - 3b)$

Solution:

(i) $(a + 7) \times (b - 5) = a \times (b - 5) + 7 \times (b - 5)$
$= ab - 5a + 7b - 35$

Note that there are no like terms involved in this multiplication.

(ii) $(a^2 + 2b^2) \times (5a - 3b) = a^2 (5a - 3b) + 2b^2 \times (5a - 3b)$
$= 5a^3 - 3a^2b + 10ab^2 - 6b^3$

8.5.2 Multiplying a binomial by a trinomial

In this multiplication, we shall have to multiply each of the three terms in the trinomial by each of the two terms in the binomial. We shall get in all $3 \times 2 = 6$ terms, which may reduce to 5 or less, if the term by term multiplication results in like terms. Consider

$\underbrace{(a + 7)}_{\text{binomial}} \times \underbrace{(a^2 + 3a + 5)}_{\text{trinomial}} = a \times (a^2 + 3a + 5) + 7 \times (a^2 + 3a + 5)$ [using the distributive law]
$= a^3 + 3a^2 + 5a + 7a^2 + 21a + 35$
$= a^3 + (3a^2 + 7a^2) + (5a + 21a) + 35$
$= a^3 + 10a^2 + 26a + 35$ (Why are there only 4 terms in the final result?)

Example 10: Simplify $(a + b)(2a - 3b + c) - (2a - 3b)c$.

Solution: We have

$(a + b)(2a - 3b + c) = a(2a - 3b + c) + b(2a - 3b + c)$
$= 2a^2 - 3ab + ac + 2ab - 3b^2 + bc$
$= 2a^2 - ab - 3b^2 + bc + ac$ (Note, $-3ab$ and $2ab$ are like terms)

and $(2a - 3b)c = 2ac - 3bc$

Therefore,

$(a + b)(2a - 3b + c) - (2a - 3b)c = 2a^2 - ab - 3b^2 + bc + ac - (2ac - 3bc)$
$= 2a^2 - ab - 3b^2 + bc + ac - 2ac + 3bc$
$= 2a^2 - ab - 3b^2 + (bc + 3bc) + (ac - 2ac)$
$= 2a^2 - 3b^2 - ab + 4bc - ac$

© NCERT not to be republished

EXERCISE 8.4

1. Multiply the binomials.

(i) $(2x + 5)$ and $(4x – 3)$ (ii) $(y – 8)$ and $(3y – 4)$

(iii) $(2.5l – 0.5m)$ and $(2.5l + 0.5m)$ (iv) $(a + 3b)$ and $(x + 5)$

(v) $(2pq + 3q^2)$ and $(3pq – 2q^2)$

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right)$ and $4\left(a^2 – \frac{2}{3}b^2\right)$

2. Find the product.

(i) $(5 – 2x)(3 + x)$ (ii) $(x + 7y)(7x – y)$

(iii) $(a^2 + b)(a + b^2)$ (iv) $(p^2 – q^2)(2p + q)$

3. Simplify.

(i) $(x^2 – 5)(x + 5) + 25$ (ii) $(a^2 + 5)(b^3 + 3) + 5$

(iii) $(t + s^2)(t^2 – s)$

(iv) $(a + b)(c – d) + (a – b)(c + d) + 2(ac + bd)$

(v) $(x + y)(2x + y) + (x + 2y)(x – y)$ (vi) $(x + y)(x^2 – xy + y^2)$

(vii) $(1.5x – 4y)(1.5x + 4y + 3) – 4.5x + 12y$

(viii) $(a + b + c)(a + b – c)$

WHAT HAVE WE DISCUSSED?

1. Expressions are formed from **variables** and **constants**.
2. Terms are added to form **expressions**. Terms themselves are formed as product of **factors**.
3. Expressions that contain exactly one, two and three terms are called **monomials**, **binomials** and **trinomials** respectively. In general, any expression containing one or more terms with non-zero coefficients (and with variables having non- negative integers as exponents) is called a **polynomial**.
4. **Like** terms are formed from the same variables and the powers of these variables are the same, too. Coefficients of like terms need not be the same.
5. While adding (or subtracting) polynomials, first look for like terms and add (or subtract) them; then handle the unlike terms.
6. There are number of situations in which we need to multiply algebraic expressions: for example, in finding area of a rectangle, the sides of which are given as expressions.
7. A monomial multiplied by a monomial always gives a monomial.
8. While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.
9. In carrying out the multiplication of a polynomial by a binomial (or trinomial), we multiply term by term, i.e., every term of the polynomial is multiplied by every term in the binomial (or trinomial). Note that in such multiplication, we may get terms in the product which are like and have to be combined.

© NCERT not to be republished

CHAPTER

9

Mensuration

0852CH11

9.1 Introduction

We have learnt that for a closed plane figure, the perimeter is the distance around its boundary and its area is the region covered by it. We found the area and perimeter of various plane figures such as triangles, rectangles, circles etc. We have also learnt to find the area of pathways or borders in rectangular shapes.

In this chapter, we will try to solve problems related to perimeter and area of other plane closed figures like quadrilaterals.

We will also learn about surface area and volume of solids such as cube, cuboid and cylinder.

9.2 Area of a Polygon

We split a quadrilateral into triangles and find its area. Similar methods can be used to find the area of a polygon. Observe the following for a pentagon: (Fig 9.1, 9.2)

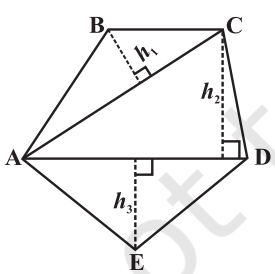


Fig 9.1

By constructing two diagonals AC and AD the pentagon ABCDE is divided into three parts. So, area ABCDE = area of Δ ABC + area of Δ ACD + area of Δ AED.

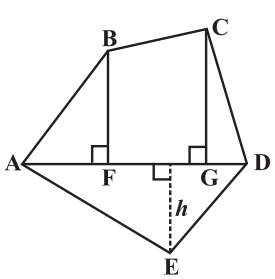


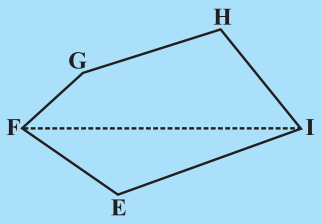
Fig 9.2

By constructing one diagonal AD and two perpendiculars BF and CG on it, pentagon ABCDE is divided into four parts. So, area of ABCDE = area of right angled Δ AFB + area of trapezium BFGC + area of right angled Δ CGD + area of Δ AED. (Identify the parallel sides of trapezium BFGC.)

© NCERT not to be republished

TRY THESE

(i) Divide the following polygons (Fig 9.3) into parts (triangles and trapezium) to find out its area.



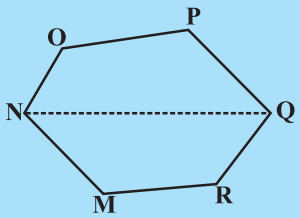


Fig 9.3

FI is a diagonal of polygon EFGHI NQ is a diagonal of polygon MNOPQR

(ii) Polygon ABCDE is divided into parts as shown below (Fig 9.4). Find its area if AD = 8 cm, AH = 6 cm, AG = 4 cm, AF = 3 cm and perpendiculars BF = 2 cm, CH = 3 cm, EG = 2.5 cm.

Area of Polygon ABCDE = area of Δ AFB + ...

Area of Δ AFB $= \frac{1}{2} \times AF \times BF = \frac{1}{2} \times 3 \times 2 = ...$

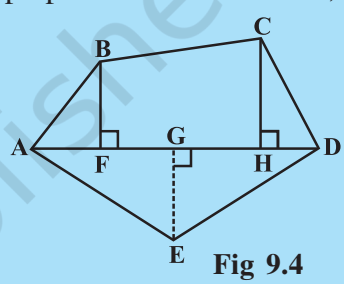


Fig 9.4

Area of trapezium FBCH $= FH \times \frac{(BF + CH)}{2}$

$= 3 \times \frac{(2+3)}{2}$ [FH = AH – AF]

Area of ΔCHD $= \frac{1}{2} \times HD \times CH = ...$; Area of ΔADE $= \frac{1}{2} \times AD \times GE = ...$

So, the area of polygon ABCDE = ...

(iii) Find the area of polygon MNOPQR (Fig 9.5) if MP = 9 cm, MD = 7 cm, MC = 6 cm, MB = 4 cm, MA = 2 cm

NA, OC, QD and RB are perpendiculars to diagonal MP.

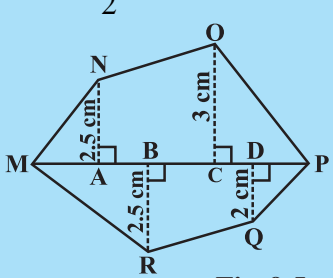


Fig 9.5

Example 1: The area of a trapezium shaped field is 480 m^2, the distance between two parallel sides is 15 m and one of the parallel side is 20 m. Find the other parallel side.

Solution: One of the parallel sides of the trapezium is a = 20 m, let another parallel side be b, height h = 15 m.

The given area of trapezium = 480 m^2.

Area of a trapezium $= \frac{1}{2} h (a + b)$

So $480 = \frac{1}{2} \times 15 \times (20 + b)$ or $\frac{480 \times 2}{15} = 20 + b$

or $64 = 20 + b$ or $b = 44$ m

Hence the other parallel side of the trapezium is 44 m.

© NCERT not to be republished

Example 2: The area of a rhombus is 240 cm^2 and one of the diagonals is 16 cm. Find the other diagonal.

Solution: Let length of one diagonal $d_1 = 16$ cm

and length of the other diagonal $= d_2$

$$\text{Area of the rhombus} = \frac{1}{2}\, d_1 \,.\, d_2 = 240$$

So, $$\frac{1}{2} 16 \cdot d_2 = 240$$

Therefore, $$d_2 = 30 \text{ cm}$$

Hence the length of the second diagonal is 30 cm.

Example 3: There is a hexagon MNOPQR of side 5 cm (Fig 9.6). Aman and Ridhima divided it in two different ways (Fig 9.7).

Find the area of this hexagon using both ways.

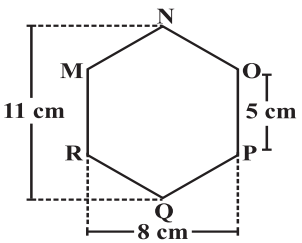
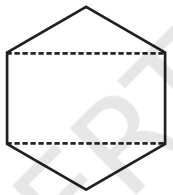
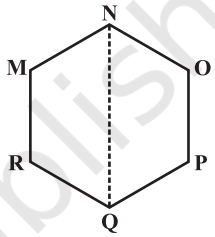


Fig 9.6



Ridhima's method



Aman's method

Fig 9.7

Solution: Aman's method:

Since it is a hexagon so NQ divides the hexagon into two congruent trapeziums. You can verify it by paper folding (Fig 9.8).

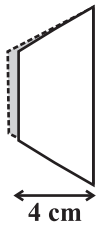


Fig 9.8

Now area of trapezium MNQR $= 4 \times \frac{(11+5)}{2} = 2 \times 16 = 32$ cm^2.

So the area of hexagon MNOPQR $= 2 \times 32 = 64$ cm^2.

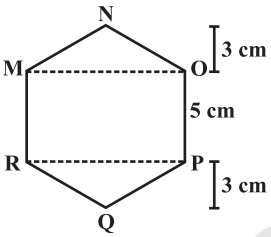


Fig 9.9

Ridhima's method:

Δ MNO and Δ RPQ are congruent triangles with altitude 3 cm (Fig 9.9).

You can verify this by cutting off these two triangles and placing them on one another.

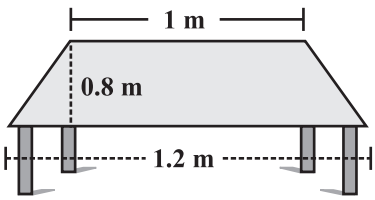
Area of Δ MNO $= \frac{1}{2} \times 8 \times 3 = 12$ cm^2 = Area of Δ RPQ

Area of rectangle MOPR $= 8 \times 5 = 40$ cm^2.

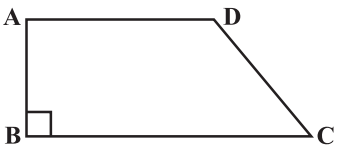
Now, area of hexagon MNOPQR $= 40 + 12 + 12 = 64$ cm^2.

EXERCISE 9.1

1. The shape of the top surface of a table is a trapezium. Find its area if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.

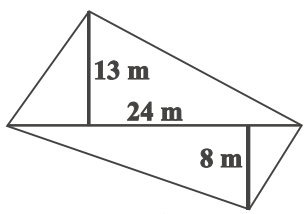


© NCERT not to be republished



2. The area of a trapezium is 34 cm^2 and the length of one of the parallel sides is 10 cm and its height is 4 cm. Find the length of the other parallel side.

3. Length of the fence of a trapezium shaped field ABCD is 120 m. If BC = 48 m, CD = 17 m and AD = 40 m, find the area of this field. Side AB is perpendicular to the parallel sides AD and BC.

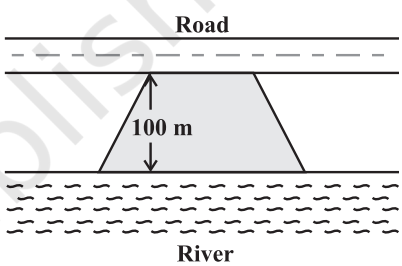


4. The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. Find the area of the field.

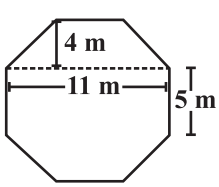
5. The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.

6. Find the area of a rhombus whose side is 5 cm and whose altitude is 4.8 cm. If one of its diagonals is 8 cm long, find the length of the other diagonal.

7. The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m^2 is ₹ 4.



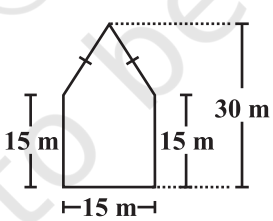
8. Mohan wants to buy a trapezium shaped field. Its side along the river is parallel to and twice the side along the road. If the area of this field is 10500 m^2 and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river.

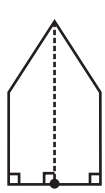


9. Top surface of a raised platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.

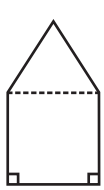
10. There is a pentagonal shaped park as shown in the figure.

For finding its area Jyoti and Kavita divided it in two different ways.

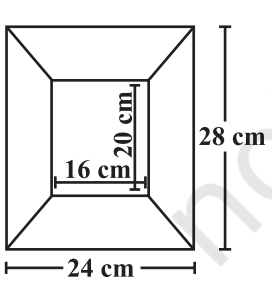




Joyti's diagram



Kavita's diagram



Find the area of this park using both ways. Can you suggest some other way of finding its area?

11. Diagram of the adjacent picture frame has outer dimensions = 24 cm × 28 cm and inner dimensions 16 cm × 20 cm. Find the area of each section of the frame, if the width of each section is same.

9.3 Solid Shapes

In your earlier classes you have studied that two dimensional figures can be identified as the faces of three dimensional shapes. Observe the solids which we have discussed so far (Fig 9.10).

© NCERT not to be republished

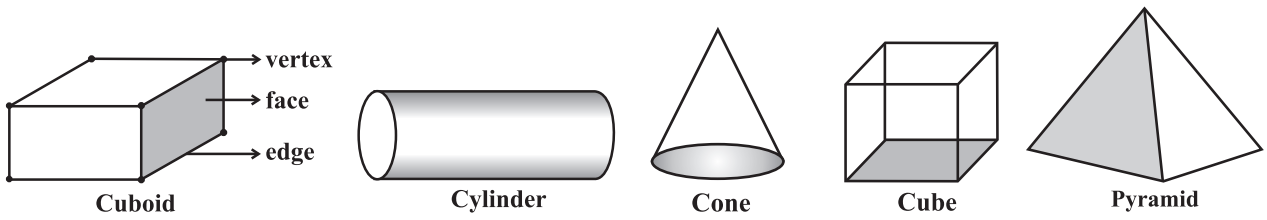


Fig 9.10

Observe that some shapes have two or more than two identical (congruent) faces. Name them. Which solid has all congruent faces?

DO THIS

Soaps, toys, pastes, snacks etc. often come in the packing of cuboidal, cubical or cylindrical boxes. Collect, such boxes (Fig 9.11).

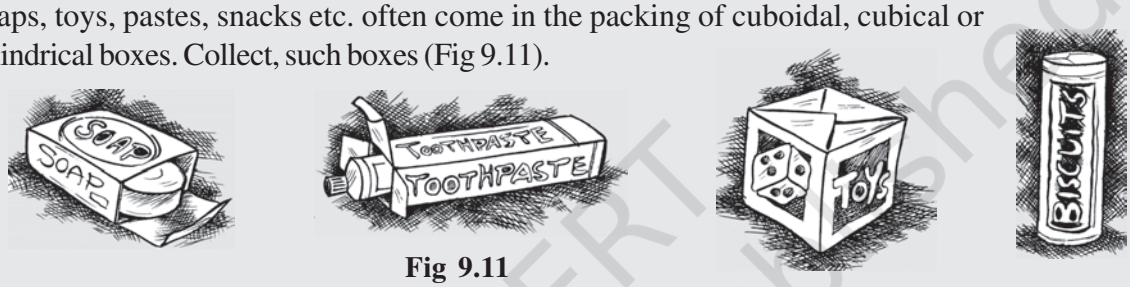
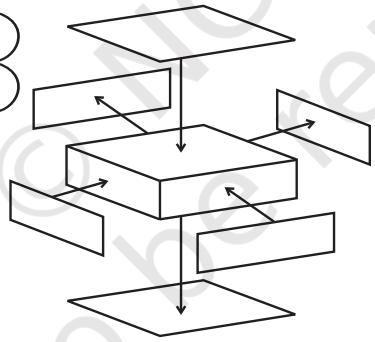


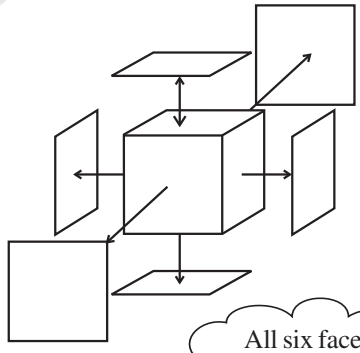
Fig 9.11

Cuboidal Box

All six faces are rectangular, and opposites faces are identical. So there are three pairs of identical faces.



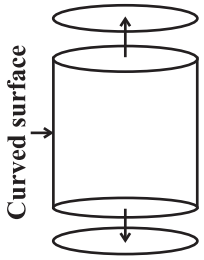
Cubical Box



All six faces are squares and identical.

Cylindrical Box

One curved surface and two circular faces which are identical.



Now take one type of box at a time. Cut out all the faces it has. Observe the shape of each face and find the number of faces of the box that are identical by placing them on each other. Write down your observations.

© NCERT not to be republished

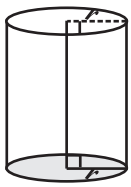


Fig 9.12
(This is a right circular cylinder)

Did you notice the following:

The cylinder has congruent circular faces that are parallel to each other (Fig 9.12). Observe that the line segment joining the center of circular faces is perpendicular to the base. Such cylinders are known as **right circular cylinders**. We are only going to study this type of cylinders, though there are other types of cylinders as well (Fig 9.13).

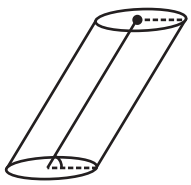
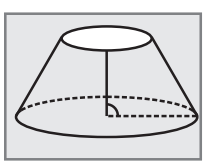


Fig 9.13
(This is not a right circular cylinder)

THINK, DISCUSS AND WRITE



Why is it incorrect to call the solid shown here a cylinder?

9.4 Surface Area of Cube, Cuboid and Cylinder

Imran, Monica and Jaspal are painting a cuboidal, cubical and a cylindrical box respectively of same height (Fig 9.4).

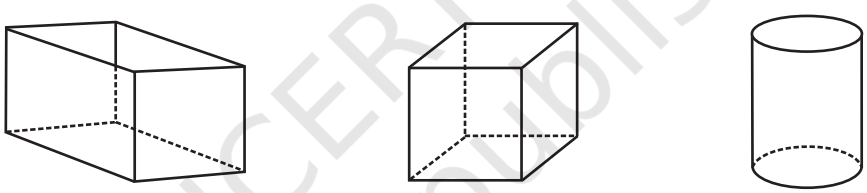


Fig 9.4

They try to determine who has painted more area. Hari suggested that finding the surface area of each box would help them find it out.

To find the total surface area, find the area of each face and then add. The surface area of a solid is the sum of the areas of its faces. To clarify further, we take each shape one by one.

9.4.1 Cuboid

Suppose you cut open a cuboidal box and lay it flat (Fig 9.15). We can see a net as shown below (Fig 9.16).

Write the dimension of each side. You know that a cuboid has three pairs of identical faces. What expression can you use to find the area of each face?

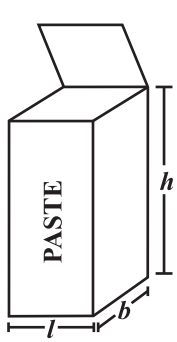


Fig 9.15

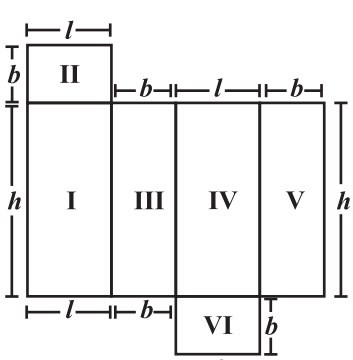


Fig 9.16

Find the total area of all the faces of the box. We see that the total surface area of a cuboid is area I + area II + area III + area IV +area V + area VI

$= h \times l + b \times l + b \times h + l \times h + b \times h + l \times b$

© NCERT not to be republished

So total surface area = 2 $(h \times l + b \times h + b \times l) = 2(lb + bh + hl)$

where h, l and b are the height, length and width of the cuboid respectively.

Suppose the height, length and width of the box shown above are 20 cm, 15 cm and 10 cm respectively.

Then the total surface area = 2 $(20 \times 15 + 20 \times 10 + 10 \times 15)$

$= 2 (300 + 200 + 150) = 1300 \text{ m}^2$.

TRY THESE

Find the total surface area of the following cuboids (Fig 9.17):

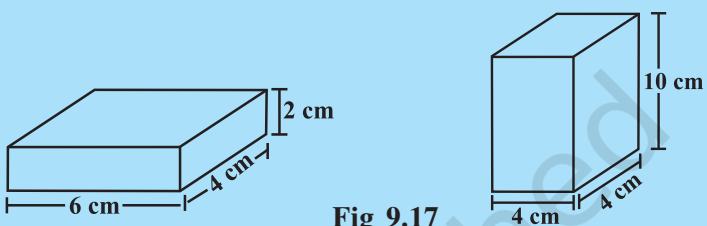


Fig 9.17

- The side walls (the faces excluding the top and bottom) make the lateral surface area of the cuboid. For example, the total area of all the four walls of the cuboidal room in which you are sitting is the lateral surface area of this room (Fig 9.18). Hence, the lateral surface area of a cuboid is given by $2(h \times l + b \times h)$ or $2h\,(l + b)$.

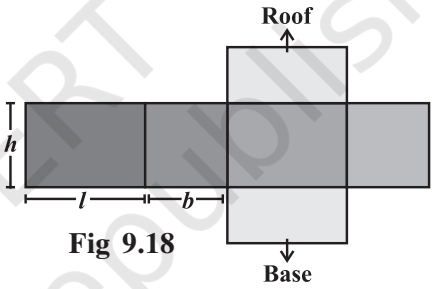


Fig 9.18

DO THIS

(i) Cover the lateral surface of a cuboidal duster (which your teacher uses in the class room) using a strip of brown sheet of paper, such that it just fits around the surface. Remove the paper. Measure the area of the paper. Is it the lateral surface area of the duster?

(ii) Measure length, width and height of your classroom and find

(a) the total surface area of the room, ignoring the area of windows and doors.

(b) the lateral surface area of this room.

(c) the total area of the room which is to be white washed.

THINK, DISCUSS AND WRITE

1. Can we say that the total surface area of cuboid = lateral surface area + 2 × area of base?
2. If we interchange the lengths of the base and the height of a cuboid (Fig 9.19(i)) to get another cuboid (Fig 9.19(ii)), will its lateral surface area change?

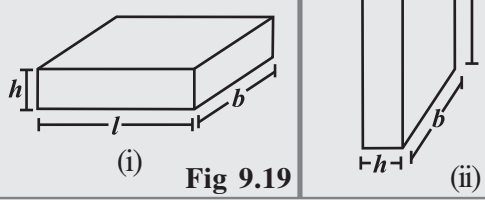


Fig 9.19

© NCERT not to be republished

9.4.2 Cube

DO THIS

Draw the pattern shown on a squared paper and cut it out [Fig 9.20(i)]. (You know that this pattern is a net of a cube. Fold it along the lines [Fig 9.20(ii)] and tape the edges to form a cube [Fig 9.20(iii)].

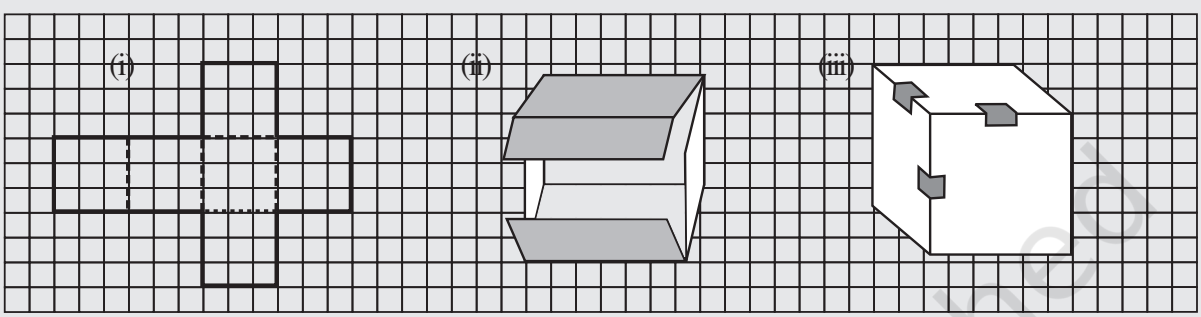


Fig 9.20

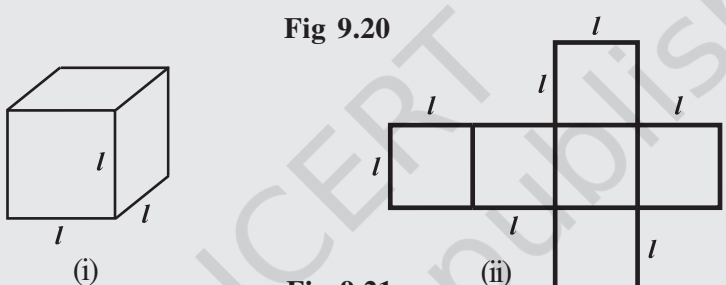


Fig 9.21

(a) What is the length, width and height of the cube? Observe that all the faces of a cube are square in shape. This makes length, height and width of a cube equal (Fig 9.21(i)).

(b) Write the area of each of the faces. Are they equal?

(c) Write the total surface area of this cube.

(d) If each side of the cube is l, what will be the area of each face? (Fig 9.21(ii)).

Can we say that the total surface area of a cube of side l is $6l^2$?

TRY THESE

Find the surface area of cube A and lateral surface area of cube B (Fig 9.22).

Fig 9.22

© NCERT not to be republished

THINK, DISCUSS AND WRITE

(i) Two cubes each with side b are joined to form a cuboid (Fig 9.23). What is the surface area of this cuboid? Is it $12b^2$? Is the surface area of cuboid formed by joining three such cubes, $18b^2$? Why?

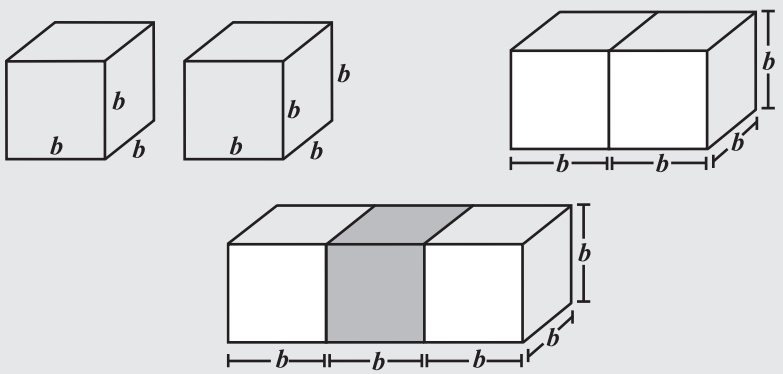


Fig 9.23

(ii) How will you arrange 12 cubes of equal length to form a cuboid of smallest surface area?

(iii) After the surface area of a cube is painted, the cube is cut into 64 smaller cubes of same dimensions (Fig 9.24). How many have no face painted? 1 face painted? 2 faces painted? 3 faces painted?

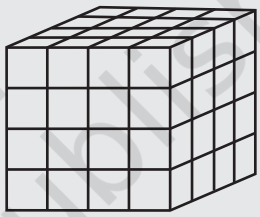


Fig 9.24

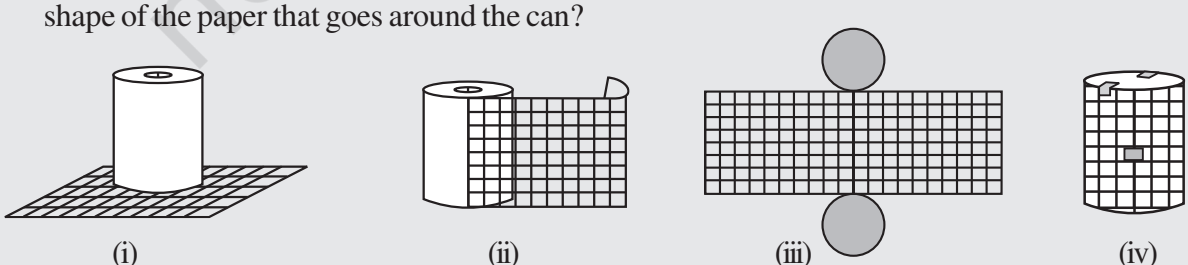
9.4.3 Cylinders

Most of the cylinders we observe are right circular cylinders. For example, a tin, round pillars, tube lights, water pipes etc.

DO THIS

(i) Take a cylindrical can or box and trace the base of the can on graph paper and cut it [Fig 9.25(i)]. Take another graph paper in such a way that its width is equal to the height of the can. Wrap the strip around the can such that it just fits around the can (remove the excess paper) [Fig 9.25(ii)].

Tape the pieces [Fig 9.25(iii)] together to form a cylinder [Fig 9.25(iv)]. What is the shape of the paper that goes around the can?



(i) (ii) (iii) (iv)

Fig 9.25

© NCERT not to be republished

Of course it is rectangular in shape. When you tape the parts of this cylinder together, the length of the rectangular strip is equal to the circumference of the circle. Record the radius (r) of the circular base, length (l) and width (h) of the rectangular strip. Is $2\pi r$ = length of the strip. Check if the area of rectangular strip is $2\pi rh$. Count how many square units of the squared paper are used to form the cylinder. Check if this count is approximately equal to $2\pi r(r+h)$.

(ii) We can deduce the relation $2\pi r(r+h)$ as the surface area of a cylinder in another way. Imagine cutting up a cylinder as shown below (Fig 9.26).

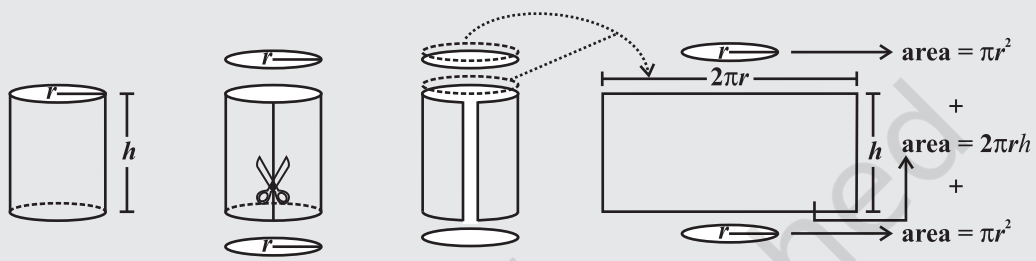


Fig 9.26

Note: We take π to be $\frac{22}{7}$ unless otherwise stated.

The lateral (or curved) surface area of a cylinder is $2\pi rh$.

The total surface area of a cylinder $= \pi r^2 + 2\pi rh + \pi r^2$

$= 2\pi r^2 + 2\pi rh$ or $2\pi r(r+h)$

TRY THESE

Find total surface area of the following cylinders (Fig 9.27)

Fig 9.27

THINK, DISCUSS AND WRITE

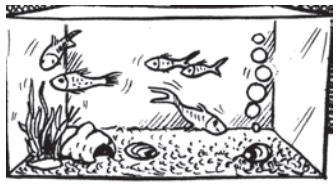
Note that lateral surface area of a cylinder is the circumference of base × height of cylinder. Can we write lateral surface area of a cuboid as perimeter of base × height of cuboid?

Example 4: An aquarium is in the form of a cuboid whose external measures are 80 cm × 30 cm × 40 cm. The base, side faces and back face are to be covered with a coloured paper. Find the area of the paper needed?

Solution: The length of the aquarium = l = 80 cm

Width of the aquarium = b = 30 cm

© NCERT not to be republished



Height of the aquarium = h = 40 cm

Area of the base = $l \times b = 80 \times 30 = 2400$ cm^2

Area of the side face = $b \times h = 30 \times 40 = 1200$ cm^2

Area of the back face = $l \times h = 80 \times 40 = 3200$ cm^2

Required area = Area of the base + area of the back face + (2 × area of a side face)

$= 2400 + 3200 + (2 \times 1200) = 8000$ cm^2

Hence the area of the coloured paper required is 8000 cm^2.

Example 5: The internal measures of a cuboidal room are 12 m × 8 m × 4 m. Find the total cost of whitewashing all four walls of a room, if the cost of white washing is ₹ 5 per m^2. What will be the cost of white washing if the ceiling of the room is also whitewashed.

Solution: Let the length of the room = l = 12 m

Width of the room = b = 8 m

Height of the room = h = 4 m

Area of the four walls of the room = Perimeter of the base × Height of the room

$= 2(l + b) \times h = 2(12 + 8) \times 4$

$= 2 \times 20 \times 4 = 160$ m^2.

Cost of white washing per m^2 = ₹ 5

Hence the total cost of white washing four walls of the room = ₹ (160 × 5) = ₹ 800

Area of ceiling is 12 × 8 = 96 m^2

Cost of white washing the ceiling = ₹ (96 × 5) = ₹ 480

So the total cost of white washing = ₹ (800 + 480) = ₹ 1280

Example 6: In a building there are 24 cylindrical pillars. The radius of each pillar is 28 cm and height is 4 m. Find the total cost of painting the curved surface area of all pillars at the rate of ₹ 8 per m^2.

Solution: Radius of cylindrical pillar, r = 28 cm = 0.28 m

height, h = 4 m

curved surface area of a cylinder = $2\pi rh$

curved surface area of a pillar = $2 \times \frac{22}{7} \times 0.28 \times 4 = 7.04$ m^2

curved surface area of 24 such pillar = 7.04 × 24 = 168.96 m^2

cost of painting an area of 1 m^2 = ₹ 8

Therefore, cost of painting 1689.6 m^2 = 168.96 × 8 = ₹ 1351.68

Example 7: Find the height of a cylinder whose radius is 7 cm and the total surface area is 968 cm^2.

Solution: Let height of the cylinder = h, radius = r = 7cm

Total surface area = $2\pi r(h + r)$

© NCERT not to be republished

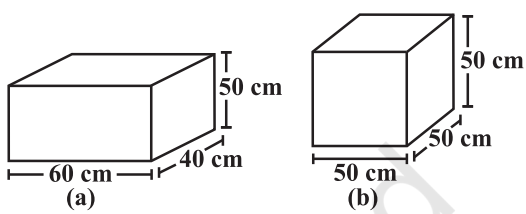
i.e., $2 \times \frac{22}{7} \times 7 \times (7 + h) = 968$

$h = 15$ cm

Hence, the height of the cylinder is 15 cm.

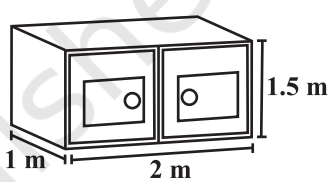
EXERCISE 9.2

1. There are two cuboidal boxes as shown in the adjoining figure. Which box requires the lesser amount of material to make?



2. A suitcase with measures 80 cm × 48 cm × 24 cm is to be covered with a tarpaulin cloth. How many metres of tarpaulin of width 96 cm is required to cover 100 such suitcases?

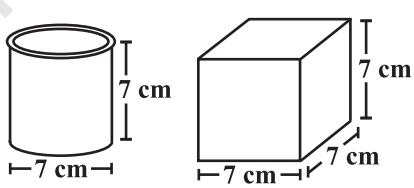
3. Find the side of a cube whose surface area is 600 cm^2.



4. Rukhsar painted the outside of the cabinet of measure 1 m × 2 m × 1.5 m. How much surface area did she cover if she painted all except the bottom of the cabinet.

5. Daniel is painting the walls and ceiling of a cuboidal hall with length, breadth and height of 15 m, 10 m and 7 m respectively. From each can of paint 100 m^2 of area is painted.

How many cans of paint will she need to paint the room?



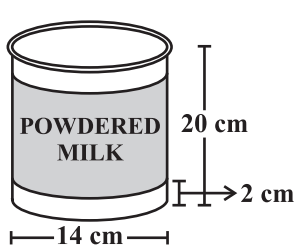
6. Describe how the two figures at the right are alike and how they are different. Which box has larger lateral surface area?

7. A closed cylindrical tank of radius 7 m and height 3 m is made from a sheet of metal. How much sheet of metal is required?

8. The lateral surface area of a hollow cylinder is 4224 cm^2. It is cut along its height and formed a rectangular sheet of width 33 cm. Find the perimeter of rectangular sheet?

9. A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road if the diameter of a road roller is 84 cm and length is 1 m.

10. A company packages its milk powder in cylindrical container whose base has a diameter of 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in the figure). If the label is placed 2 cm from top and bottom, what is the area of the label.



© NCERT not to be republished

9.5 Volume of Cube, Cuboid and Cylinder

Amount of space occupied by a three dimensional object is called its **volume**. Try to compare the volume of objects surrounding you. For example, volume of a room is greater than the volume of an almirah kept inside it. Similarly, volume of your pencil box is greater than the volume of the pen and the eraser kept inside it. Can you measure volume of either of these objects?

Remember, we use square units to find the area of a region. Here we will use cubic units to find the volume of a solid, as cube is the most convenient solid shape (just as square is the most convenient shape to measure area of a region).

For finding the area we divide the region into square units, similarly, to find the volume of a solid we need to divide it into cubical units.

Observe that the volume of each of the adjoining solids is 8 cubic units (Fig 9.28).

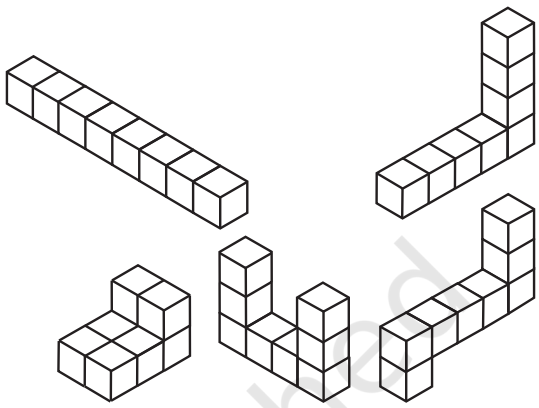


Fig 9.28

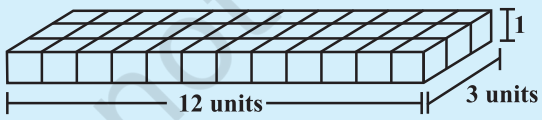
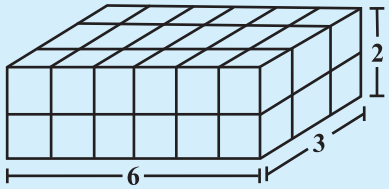
We can say that the volume of a solid is measured by counting the number of unit cubes it contains. Cubic units which we generally use to measure volume are

$$1 \text{ cubic cm} = 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^3$$
$$= 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} = \ldots\ldots\ldots\ldots \text{ mm}^3$$
$$1 \text{ cubic m} = 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^3$$
$$= \ldots\ldots\ldots\ldots\ldots\ldots \text{ cm}^3$$
$$1 \text{ cubic mm} = 1 \text{ mm} \times 1 \text{ mm} \times 1 \text{ mm} = 1 \text{ mm}^3$$
$$= 0.1 \text{ cm} \times 0.1 \text{ cm} \times 0.1 \text{ cm} = \ldots\ldots\ldots\ldots\ldots\ldots \text{ cm}^3$$

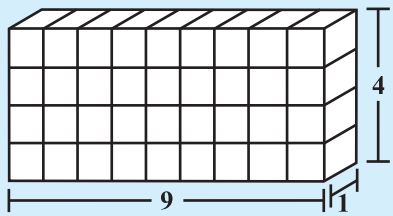
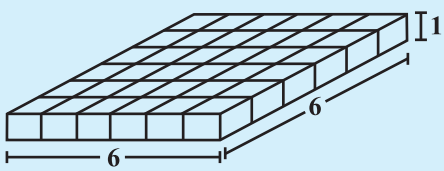
We now find some expressions to find volume of a cuboid, cube and cylinder. Let us take each solid one by one.

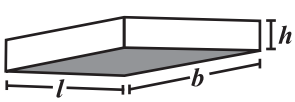
9.5.1 Cuboid

Take 36 cubes of equal size (i.e., length of each cube is same). Arrange them to form a cuboid. You can arrange them in many ways. Observe the following table and fill in the blanks.

	cuboid	length	breadth	height	$l \times b \times h = V$
(i)		12	3	1	$12 \times 3 \times 1 = 36$
(ii)		...	...	...	...

© NCERT not to be republished

(iii)		...	...	...	...
(iv)		...	...	...	...



What do you observe?

Since we have used 36 cubes to form these cuboids, volume of each cuboid is 36 cubic units. Also volume of each cuboid is equal to the product of length, breadth and height of the cuboid. From the above example we can say volume of cuboid $= l \times b \times h$. Since $l \times b$ is the area of its base we can also say that,

Volume of cuboid = area of the base × height

DO THIS

Take a sheet of paper. Measure its area. Pile up such sheets of paper of same size to make a cuboid (Fig 9.29). Measure the height of this pile. Find the volume of the cuboid by finding the product of the area of the sheet and the height of this pile of sheets.

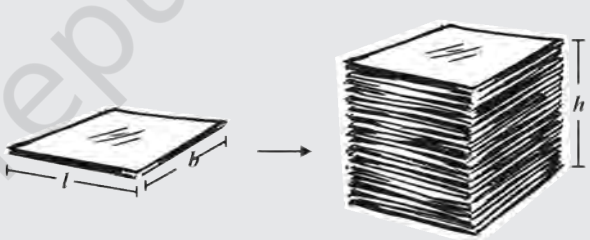
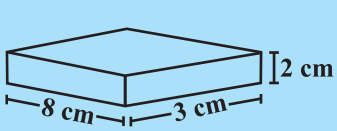


Fig 9.29

This activity illustrates the idea that volume of a solid can be deduced by this method also (if the base and top of the solid are congruent and parallel to each other and its edges are perpendicular to the base). Can you think of such objects whose volume can be found by using this method?

TRY THESE

Find the volume of the following cuboids (Fig 9.30).

(i) 

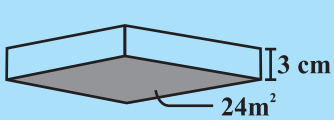


Fig 9.30

© NCERT not to be republished

9.5.2 Cube

The cube is a special case of a cuboid, where $l = b = h$.
Hence, volume of cube $= l \times l \times l = l^3$

TRY THESE

Find the volume of the following cubes

(a) with a side 4 cm (b) with a side 1.5 m

DO THIS

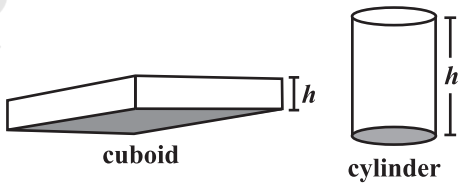
Arrange 64 cubes of equal size in as many ways as you can to form a cuboid. Find the surface area of each arrangement. Can solid shapes of same volume have same surface area?

THINK, DISCUSS AND WRITE

A company sells biscuits. For packing purpose they are using cuboidal boxes: box A $\rightarrow$ 3 cm $\times$ 8 cm $\times$ 20 cm, box B $\rightarrow$ 4 cm $\times$ 12 cm $\times$ 10 cm. What size of the box will be economical for the company? Why? Can you suggest any other size (dimensions) which has the same volume but is more economical than these?

9.5.3 Cylinder

We know that volume of a cuboid can be found by finding the product of area of base and its height. Can we find the volume of a cylinder in the same way?



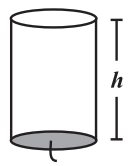
Just like cuboid, cylinder has got a top and a base which are congruent and parallel to each other. Its lateral surface is also perpendicular to the base, just like cuboid.

So the Volume of a cuboid = area of base $\times$ height

$= l \times b \times h = lbh$

Volume of cylinder = area of base $\times$ height

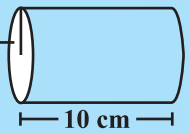
$= \pi r^2 \times h = \pi r^2 h$



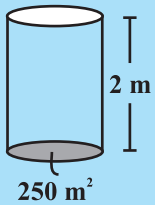
TRY THESE

Find the volume of the following cylinders.

(i)



(ii)



© NCERT not to be republished

9.6 Volume and Capacity

There is not much difference between these two words.

(a) Volume refers to the amount of space occupied by an object.

(b) Capacity refers to the quantity that a container holds.

Note: If a water tin holds 100 cm^3 of water then the capacity of the water tin is 100 cm^3.

Capacity is also measured in terms of litres. The relation between litre and cm^3 is, 1 mL = 1 cm^3, 1 L = 1000 cm^3. Thus, 1 m^3 = 1000000 cm^3 = 1000 L.

Example 8: Find the height of a cuboid whose volume is 275 cm^3 and base area is 25 cm^2.

Solution: Volume of a cuboid = Base area × Height

$$\text{Hence height of the cuboid} = \frac{\text{Volume of cuboid}}{\text{Base area}}$$

$$= \frac{275}{25} = 11 \text{ cm}$$

Height of the cuboid is 11 cm.

Example 9: A godown is in the form of a cuboid of measures 60 m × 40 m × 30 m. How many cuboidal boxes can be stored in it if the volume of one box is 0.8 m^3 ?

Solution: Volume of one box = 0.8 m^3

Volume of godown = $60 \times 40 \times 30 = 72000$ m^3

$$\text{Number of boxes that can be stored in the godown} = \frac{\text{Volume of the godown}}{\text{Volume of one box}}$$

$$= \frac{60 \times 40 \times 30}{0.8} = 90{,}000$$

Hence the number of cuboidal boxes that can be stored in the godown is 90,000.

Example 10: A rectangular paper of width 14 cm is rolled along its width and a cylinder of radius 20 cm is formed. Find the volume of the cylinder (Fig 9.31). (Take $\frac{22}{7}$ for π)

Solution: A cylinder is formed by rolling a rectangle about its width. Hence the width of the paper becomes height and radius of the cylinder is 20 cm.

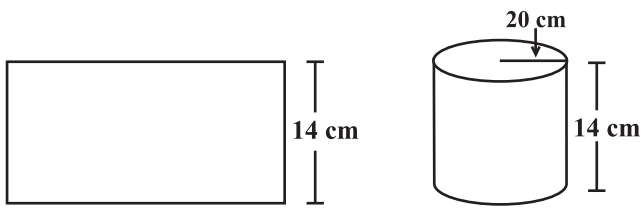


Fig 9.31

Height of the cylinder = h = 14 cm

Radius = r = 20 cm

© NCERT not to be republished

Volume of the cylinder = V = $\pi r^2 h$

$$= \frac{22}{7} \times 20 \times 20 \times 14 = 17600 \text{ cm}^3$$

Hence, the volume of the cylinder is 17600 cm^3.

Example 11: A rectangular piece of paper 11 cm × 4 cm is folded without overlapping to make a cylinder of height 4 cm. Find the volume of the cylinder.

Solution: Length of the paper becomes the perimeter of the base of the cylinder and width becomes height.

Let radius of the cylinder = r and height = h

Perimeter of the base of the cylinder = $2\pi r = 11$

or $$2 \times \frac{22}{7} \times r = 11$$

Therefore, $$r = \frac{7}{4} \text{ cm}$$

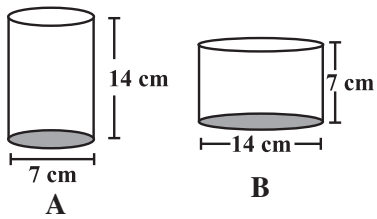
Volume of the cylinder = V = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 4 \text{ cm}^3 = 38.5 \text{ cm}^3.$$

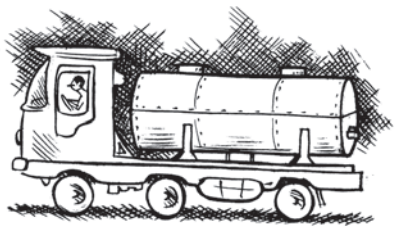
Hence the volume of the cylinder is 38.5 cm^3.

EXERCISE 9.3

1. Given a cylindrical tank, in which situation will you find surface area and in which situation volume.
   (a) To find how much it can hold.
   (b) Number of cement bags required to plaster it.
   (c) To find the number of smaller tanks that can be filled with water from it.
2. Diameter of cylinder A is 7 cm, and the height is 14 cm. Diameter of cylinder B is 14 cm and height is 7 cm. Without doing any calculations can you suggest whose volume is greater? Verify it by finding the volume of both the cylinders. Check whether the cylinder with greater volume also has greater surface area?



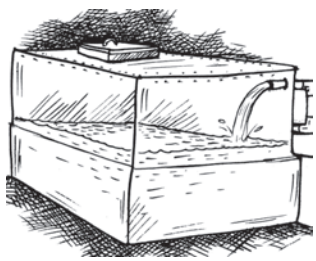
3. Find the height of a cuboid whose base area is 180 cm^2 and volume is 900 cm^3?
4. A cuboid is of dimensions 60 cm × 54 cm × 30 cm. How many small cubes with side 6 cm can be placed in the given cuboid?
5. Find the height of the cylinder whose volume is 1.54 m^3 and diameter of the base is 140 cm ?
6. A milk tank is in the form of cylinder whose radius is 1.5 m and length is 7 m. Find the quantity of milk in litres that can be stored in the tank?



7. If each edge of a cube is doubled,
   (i) how many times will its surface area increase?
   (ii) how many times will its volume increase?

© NCERT not to be republished

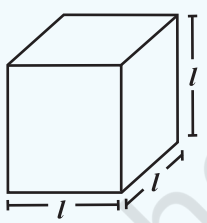
8. Water is pouring into a cubiodal reservoir at the rate of 60 litres per minute. If the volume of reservoir is 108 m^3, find the number of hours it will take to fill the reservoir.



WHAT HAVE WE DISCUSSED?

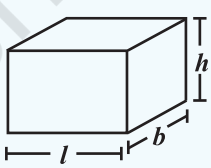
1. **Surface area** of a solid is the sum of the areas of its faces.

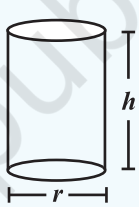
2. Surface area of
a cuboid = $2(lb + bh + hl)$
a cube = $6l^2$
a cylinder = $2\pi r(r + h)$



3. Amount of region occupied by a solid is called its **volume**.

4. Volume of
a cuboid = $l \times b \times h$
a cube = l^3
a cylinder = $\pi r^2 h$





5. (i) 1 cm^3 = 1 mL
(ii) 1L = 1000 cm^3
(iii) 1 m^3 = 1000000 cm^3 = 1000L

© NCERT not to be republished

CHAPTER 10

Exponents and Powers

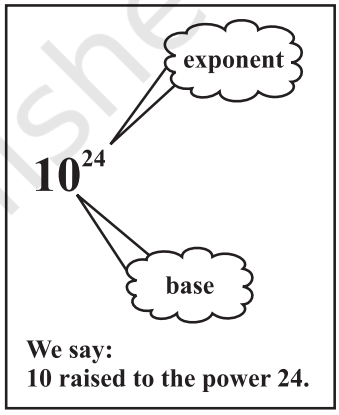
0852CH12

10.1 Introduction

Do you know?

Mass of earth is 5,970,000,000,000, 000, 000, 000, 000 kg. We have already learnt in earlier class how to write such large numbers more conveniently using exponents, as, 5.97×10^{24} kg.

We read 10^{24} as 10 raised to the power 24.



We know $2^5 = 2 \times 2 \times 2 \times 2 \times 2$

and $2^m = 2 \times 2 \times 2 \times 2 \times \ldots \times 2 \times 2 \ldots$ (m times)

Let us now find what is 2^{-2} is equal to?

10.2 Powers with Negative Exponents

You know that, $10^2 = 10 \times 10 = 100$

$$10^1 = 10 = \frac{100}{10}$$

$$10^0 = 1 = \frac{10}{10}$$

$10^{-1} = ?$

Exponent is a negative integer.

As the exponent decreases by1, the value becomes one-tenth of the previous value.

Continuing the above pattern we get, $10^{-1} = \frac{1}{10}$

Similarly $10^{-2} = \frac{1}{10} \div 10 = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = \frac{1}{10^2}$

$$10^{-3} = \frac{1}{100} \div 10 = \frac{1}{100} \times \frac{1}{10} = \frac{1}{1000} = \frac{1}{10^3}$$

What is 10^{-10} equal to?

© NCERT not to be republished

Now consider the following.

$3^3 = 3 \times 3 \times 3 = 27$

$3^2 = 3 \times 3 = 9 = \frac{27}{3}$

$3^1 = 3 = \frac{9}{3}$

$3^\circ = 1 = \frac{3}{3}$

The previous number is divided by the base 3.

So looking at the above pattern, we say

$$3^{-1} = 1 \div 3 = \frac{1}{3}$$

$$3^{-2} = \frac{1}{3} \div 3 = \frac{1}{3 \times 3} = \frac{1}{3^2}$$

$$3^{-3} = \frac{1}{3^2} \div 3 = \frac{1}{3^2} \times \frac{1}{3} = \frac{1}{3^3}$$

You can now find the value of 2^{-2} in a similar manner.

We have, $10^{-2} = \frac{1}{10^2}$ or $10^2 = \frac{1}{10^{-2}}$

$10^{-3} = \frac{1}{10^3}$ or $10^3 = \frac{1}{10^{-3}}$

$3^{-2} = \frac{1}{3^2}$ or $3^2 = \frac{1}{3^{-2}}$ etc.

In general, we can say that for any non-zero integer a, $a^{-m} = \frac{1}{a^m}$, where m is a positive integer. a^{-m} is the multiplicative inverse of a^m.

TRY THESE

Find the multiplicative inverse of the following.

(i) 2^{-4} (ii) 10^{-5} (iii) 7^{-2} (iv) 5^{-3} (v) 10^{-100}

We learnt how to write numbers like 1425 in expanded form using exponents as $1 \times 10^3 + 4 \times 10^2 + 2 \times 10^1 + 5 \times 10^\circ$.

Let us see how to express 1425.36 in expanded form in a similar way.

We have $1425.36 = 1 \times 1000 + 4 \times 100 + 2 \times 10 + 5 \times 1 + \frac{3}{10} + \frac{6}{100}$

$= 1 \times 10^3 + 4 \times 10^2 + 2 \times 10 + 5 \times 1 + 3 \times 10^{-1} + 6 \times 10^{-2}$

$10^{-1} = \frac{1}{10}$, $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$

TRY THESE

Expand the following numbers using exponents.

(i) 1025.63 (ii) 1256.249

© NCERT not to be republished

10.3 Laws of Exponents

We have learnt that for any non-zero integer a, $a^m \times a^n = a^{m+n}$, where m and n are natural numbers. Does this law also hold if the exponents are negative? Let us explore.

(i) We know that $2^{-3} = \frac{1}{2^3}$ and $2^{-2} = \frac{1}{2^2}$

$a^{-m} = \frac{1}{a^m}$ for any non-zero integer a.

Therefore, $2^{-3} \times 2^{-2} = \frac{1}{2^3} \times \frac{1}{2^2} = \frac{1}{2^3 \times 2^2} = \frac{1}{2^{3+2}} = 2^{-5}$

–5 is the sum of two exponents – 3 and – 2

(ii) Take $(-3)^{-4} \times (-3)^{-3}$

$$(-3)^{-4} \times (-3)^{-3} = \frac{1}{(-3)^4} \times \frac{1}{(-3)^3}$$

$$= \frac{1}{(-3)^4 \times (-3)^3} = \frac{1}{(-3)^{4+3}} = (-3)^{-7}$$

$(-4) + (-3) = -7$

(iii) Now consider $5^{-2} \times 5^4$

$(-2) + 4 = 2$

$$5^{-2} \times 5^4 = \frac{1}{5^2} \times 5^4 = \frac{5^4}{5^2} = 5^{4-2} = 5^{(2)}$$

In Class VII, you have learnt that for any non-zero integer a, $\frac{a^m}{a^n} = a^{m-n}$, where m and n are natural numbers and $m > n$.

(iv) Now consider $(-5)^{-4} \times (-5)^2$

$$(-5)^{-4} \times (-5)^2 = \frac{1}{(-5)^4} \times (-5)^2 = \frac{(-5)^2}{(-5)^4} = \frac{1}{(-5)^4 \times (-5)^{-2}}$$

$$= \frac{1}{(-5)^{4-2}} = (-5)^{-(2)}$$

$(-4) + 2 = -2$

In general, we can say that for any non-zero integer a, $a^m \times a^n = a^{m+n}$, where m and n are integers.

TRY THESE

Simplify and write in exponential form.

(i) $(-2)^{-3} \times (-2)^{-4}$ (ii) $p^3 \times p^{-10}$ (iii) $3^2 \times 3^{-5} \times 3^6$

On the same lines you can verify the following laws of exponents, where a and b are non zero integers and m, n are any integers.

(i) $\frac{a^m}{a^n} = a^{m-n}$ (ii) $(a^m)^n = a^{mn}$ (iii) $a^m \times b^m = (ab)^m$

(iv) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$ (v) $a^0 = 1$

These laws you have studied in Class VII for positive exponents only.

Let us solve some examples using the above Laws of Exponents.

© NCERT not to be republished

Example 1: Find the value of

(i) 2^{-3} (ii) $\frac{1}{3^{-2}}$

Solution:

(i) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ (ii) $\frac{1}{3^{-2}} = 3^2 = 3 \times 3 = 9$

Example 2: Simplify

(i) $(-4)^5 \times (-4)^{-10}$ (ii) $2^5 \div 2^{-6}$

Solution:

(i) $(-4)^5 \times (-4)^{-10} = (-4)^{(5-10)} = (-4)^{-5} = \frac{1}{(-4)^5}$ $(a^m \times a^n = a^{m+n},\ a^{-m} = \frac{1}{a^m})$

(ii) $2^5 \div 2^{-6} = 2^{5-(-6)} = 2^{11}$ $(a^m \div a^n = a^{m-n})$

Example 3: Express 4^{-3} as a power with the base 2.

Solution: We have, $4 = 2 \times 2 = 2^2$

Therefore, $(4)^{-3} = (2 \times 2)^{-3} = (2^2)^{-3} = 2^{2 \times (-3)} = 2^{-6}$ $[(a^m)^n = a^{mn}]$

Example 4: Simplify and write the answer in the exponential form.

(i) $(2^5 \div 2^8)^5 \times 2^{-5}$ (ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

(iii) $\frac{1}{8} \times (3)^{-3}$ (iv) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

Solution:

(i) $(2^5 \div 2^8)^5 \times 2^{-5} = (2^{5-8})^5 \times 2^{-5} = (2^{-3})^5 \times 2^{-5} = 2^{-15-5} = 2^{-20} = \frac{1}{2^{20}}$

(ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3} = [(-4) \times 5 \times (-5)]^{-3} = [100]^{-3} = \frac{1}{100^3}$

[using the law $a^m \times b^m = (ab)^m$, $a^{-m} = \frac{1}{a^m}$]

(iii) $\frac{1}{8} \times (3)^{-3} = \frac{1}{2^3} \times (3)^{-3} = 2^{-3} \times 3^{-3} = (2 \times 3)^{-3} = 6^{-3} = \frac{1}{6^3}$

(iv) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-1 \times 3)^4 \times \frac{5^4}{3^4} = (-1)^4 \times 3^4 \times \frac{5^4}{3^4}$

$= (-1)^4 \times 5^4 = 5^4$ $[(-1)^4 = 1]$

Example 5: Find m so that $(-3)^{m+1} \times (-3)^5 = (-3)^7$

Solution: $(-3)^{m+1} \times (-3)^5 = (-3)^7$

$(-3)^{m+1+5} = (-3)^7$

$(-3)^{m+6} = (-3)^7$

On both the sides powers have the same base different from 1 and – 1, so their exponents must be equal.

© NCERT not to be republished

Therefore, $m + 6 = 7$

or $m = 7 - 6 = 1$

$a^n = 1$ only if $n = 0$. This will work for any a.
For $a = 1$, $1^1 = 1^2 = 1^3 = 1^{-2} = \ldots = 1$ or $(1)^n = 1$ for infinitely many n.
For $a = -1$,
$(-1)^0 = (-1)^2 = (-1)^4 = (-1)^{-2} = \ldots = 1$ or $(-1)^p = 1$ for any even integer p.

Example 6: Find the value of $\left(\frac{2}{3}\right)^{-2}$.

Solution: $\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \frac{9}{4}$

$\left(\frac{2}{3}\right)^{-2} = \frac{2^{-2}}{3^{-2}} = \frac{3^2}{2^2} = \left(\frac{3}{2}\right)^2$

In general, $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Example 7: Simplify (i) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5}$

Solution:

(i) $\left\{\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2} = \left\{\frac{1^{-2}}{3^{-2}} - \frac{1^{-3}}{2^{-3}}\right\} \div \frac{1^{-2}}{4^{-2}}$

$= \left\{\frac{3^2}{1^2} - \frac{2^3}{1^3}\right\} \div \frac{4^2}{1^2} = \{9 - 8\} \div 16 = \frac{1}{16}$

(ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-5} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-5}}{5^{-5}} = \frac{5^{-7}}{5^{-5}} \times \frac{8^{-5}}{8^{-7}} = 5^{(-7)-(-5)} \times 8^{(-5)-(-7)}$

$= 5^{-2} \times 8^2 = \frac{8^2}{5^2} = \frac{64}{25}$

EXERCISE 10.1

1. Evaluate.

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

2. Simplify and express the result in power notation with positive exponent.

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$ (iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$ (v) $2^{-3} \times (-7)^{-3}$

3. Find the value of.

(i) $(3^0 + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

© NCERT not to be republished

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$ (v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

4. Evaluate (i) $\frac{8^{-1} \times 5^3}{2^{-4}}$ (ii) $(5^{-1} \times 2^{-1}) \times 6^{-1}$

5. Find the value of m for which $5^m \div 5^{-3} = 5^5$.

6. Evaluate (i) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$ (ii) $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$

7. Simplify.

(i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$ (ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

10.4 Use of Exponents to Express Small Numbers in Standard Form

Observe the following facts.

1. The distance from the Earth to the Sun is 149,600,000,000 m.
2. The speed of light is 300,000,000 m/sec.
3. Thickness of Class VII Mathematics book is 20 mm.
4. The average diameter of a Red Blood Cell is 0.000007 mm.
5. The thickness of human hair is in the range of 0.005 cm to 0.01 cm.
6. The distance of moon from the Earth is 384, 467, 000 m (approx).
7. The size of a plant cell is 0.00001275 m.
8. Average radius of the Sun is 695000 km.
9. Mass of propellant in a space shuttle solid rocket booster is 503600 kg.
10. Thickness of a piece of paper is 0.0016 cm.
11. Diameter of a wire on a computer chip is 0.000003 m.
12. The height of Mount Everest is 8848 m.

Observe that there are few numbers which we can read like 2 cm, 8848 m, 6,95,000 km. There are some large numbers like 150,000,000,000 m and some very small numbers like 0.000007 m.

Identify very large and very small numbers from the above facts and write them in the adjacent table:

Very large numbers	Very small numbers
150,000,000,000 m	0.000007 m
---	---
---	---
---	---
---	---

We have learnt how to express very large numbers in standard form in the previous class.

For example: $150,000,000,000 = 1.5 \times 10^{11}$

Now, let us try to express 0.000007 m in standard form.

© NCERT not to be republished

$$0.000007 = \frac{7}{1000000} = \frac{7}{10^6} = 7 \times 10^{-6}$$

$0.000007 \text{ m} = 7 \times 10^{-6} \text{ m}$

Similarly, consider the thickness of a piece of paper which is 0.0016 cm.

$$0.0016 = \frac{16}{10000} = \frac{1.6 \times 10}{10^4} = 1.6 \times 10 \times 10^{-4}$$
$$= 1.6 \times 10^{-3}$$

Therefore, we can say thickness of paper is 1.6×10^{-3} cm.

150000000000. (11 10 9 8 7 6 5 4 3 2 1) Decimal is moved 11 places to the left.

0.000007 (1 2 3 4 5 6) decimal is moved 6 places to the right.

Again notice
0.0016 (1 2 3) decimal is moved 3 places to the right.

TRY THESE

1. Write the following numbers in standard form.
   (i) 0.000000564 (ii) 0.0000021 (iii) 21600000 (iv) 15240000
2. Write all the facts given in the standard form.

10.4.1 Comparing very large and very small numbers

The diameter of the Sun is 1.4×10^9 m and the diameter of the Earth is 1.2756×10^7 m. Suppose you want to compare the diameter of the Earth, with the diameter of the Sun.

Diameter of the Sun = 1.4×10^9 m

Diameter of the earth = 1.2756×10^7 m

Therefore $\frac{1.4 \times 10^9}{1.2756 \times 10^7} = \frac{1.4 \times 10^{9-7}}{1.2756} = \frac{1.4 \times 100}{1.2756}$ which is approximately 100

So, the diameter of the Sun is about 100 times the diameter of the earth.

Let us compare the size of a Red Blood cell which is 0.000007 m to that of a plant cell which is 0.00001275 m.

Size of Red Blood cell = $0.000007 \text{ m} = 7 \times 10^{-6}$ m

Size of plant cell = $0.00001275 = 1.275 \times 10^{-5}$ m

Therefore, $\frac{7 \times 10^{-6}}{1.275 \times 10^{-5}} = \frac{7 \times 10^{-6-(-5)}}{1.275} = \frac{7 \times 10^{-1}}{1.275} = \frac{0.7}{1.275} = \frac{0.7}{1.3} = \frac{1}{2}$ (approx.)

So a red blood cell is half of plant cell in size.

Mass of earth is 5.97×10^{24} kg and mass of moon is 7.35×10^{22} kg. What is the total mass?

Total mass = 5.97×10^{24} kg + 7.35×10^{22} kg.
$= 5.97 \times 100 \times 10^{22} + 7.35 \times 10^{22}$
$= 597 \times 10^{22} + 7.35 \times 10^{22}$
$= (597 + 7.35) \times 10^{22}$
$= 604.35 \times 10^{22}$ kg.

When we have to add numbers in standard form, we convert them into numbers with the same exponents.

The distance between Sun and Earth is 1.496×10^{11}m and the distance between Earth and Moon is 3.84×10^8m.

During solar eclipse moon comes in between Earth and Sun.

At that time what is the distance between Moon and Sun.

© NCERT not to be republished

Distance between Sun and Earth = 1.496×10^{11}m

Distance between Earth and Moon = 3.84×10^{8}m

Distance between Sun and Moon = $1.496 \times 10^{11} - 3.84 \times 10^{8}$

$= 1.496 \times 1000 \times 10^{8} - 3.84 \times 10^{8}$

$= (1496 - 3.84) \times 10^{8}$ m $= 1492.16 \times 10^{8}$ m

Example 8: Express the following numbers in standard form.

(i) 0.000035 (ii) 4050000

Solution: (i) $0.000035 = 3.5 \times 10^{-5}$ (ii) $4050000 = 4.05 \times 10^{6}$

Example 9: Express the following numbers in usual form.

(i) 3.52×10^{5} (ii) 7.54×10^{-4} (iii) 3×10^{-5}

Solution:

(i) $3.52 \times 10^{5} = 3.52 \times 100000 = 352000$

(ii) $7.54 \times 10^{-4} = \frac{7.54}{10^{4}} = \frac{7.54}{10000} = 0.000754$

(iii) $3 \times 10^{-5} = \frac{3}{10^{5}} = \frac{3}{100000} = 0.00003$

Again we need to convert numbers in standard form into a numbers with the same exponents.

EXERCISE 10.2

1. Express the following numbers in standard form.

(i) 0.0000000000085 (ii) 0.00000000000942

(iii) 6020000000000000 (iv) 0.00000000837

(v) 31860000000

2. Express the following numbers in usual form.

(i) 3.02×10^{-6} (ii) 4.5×10^{4} (iii) 3×10^{-8}

(iv) 1.0001×10^{9} (v) 5.8×10^{12} (vi) 3.61492×10^{6}

3. Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000}$ m.

(ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.

(iii) Size of a bacteria is 0.0000005 m

(iv) Size of a plant cell is 0.00001275 m

(v) Thickness of a thick paper is 0.07 mm

4. In a stack there are 5 books each of thickness 20mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack.

WHAT HAVE WE DISCUSSED?

1. Numbers with negative exponents obey the following laws of exponents.

(a) $a^m \times a^n = a^{m+n}$ (b) $a^m \div a^n = a^{m-n}$ (c) $(a^m)^n = a^{mn}$

(d) $a^m \times b^m = (ab)^m$ (e) $a^0 = 1$ (f) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

2. Very small numbers can be expressed in standard form using negative exponents.

© NCERT not to be republished

CHAPTER 11

Direct and Inverse Proportions

0852CH13

11.1 Introduction

Mohan prepares tea for himself and his sister. He uses 300 mL of water, 2 spoons of sugar, 1 spoon of tea leaves and 50 mL of milk. How much quantity of each item will he need, if he has to make tea for five persons?

If two students take 20 minutes to arrange chairs for an assembly, then how much time would five students take to do the same job?

We come across many such situations in our day-to-day life, where we need to see variation in one quantity bringing in variation in the other quantity.

For example:

(i) If the number of articles purchased increases, the total cost also increases.

(ii) More the money deposited in a bank, more is the interest earned.

(iii) As the speed of a vehicle increases, the time taken to cover the same distance decreases.

(iv) For a given job, more the number of workers, less will be the time taken to complete the work.

Observe that change in one quantity leads to change in the other quantity.

Write five more such situations where change in one quantity leads to change in another quantity.

How do we find out the quantity of each item needed by Mohan? Or, the time five students take to complete the job?

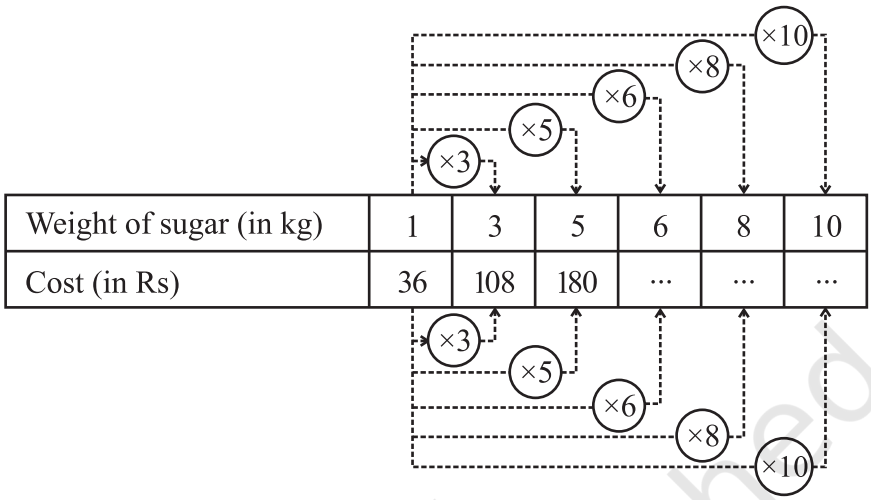
To answer such questions, we now study some concepts of variation.

11.2 Direct Proportion

If the cost of 1 kg of sugar is ₹ 36, then what would be the cost of 3 kg sugar? It is ₹ 108.

© NCERT not to be republished

Similarly, we can find the cost of 5 kg or 8 kg of sugar. Study the following table.



Observe that as weight of sugar increases, cost also increases in such a manner that their ratio remains constant.

Take one more example. Suppose a car uses 4 litres of petrol to travel a distance of 60 km. How far will it travel using 12 litres? The answer is 180 km. How did we calculate it? Since petrol consumed in the second instance is 12 litres, i.e., three times of 4 litres, the distance travelled will also be three times of 60 km. In other words, when the petrol consumption becomes three-fold, the distance travelled is also three fold the previous one. Let the consumption of petrol be x litres and the corresponding distance travelled be y km . Now, complete the following table:

Petrol in litres (x)	4	8	12	15	20	25
Distance in km (y)	60	...	180	...	...	...

We find that as the value of x increases, value of y also increases in such a way that the ratio $\frac{x}{y}$ does not change; it remains constant (say k). In this case, it is $\frac{1}{15}$ (check it!).

We say that ***x* and *y* are in direct proportion, if** $\frac{x}{y} = k$ **or** $x = ky$.

In this example, $\frac{4}{60} = \frac{12}{180}$, where 4 and 12 are the quantities of petrol consumed in litres (x) and 60 and 180 are the distances (y) in km. So when x and y are in **direct proportion**, we can write $\frac{x_1}{y_1} = \frac{x_2}{y_2}$. [$y_1$, y_2 are values of y corresponding to the values x_1, x_2 of x respectively]

The consumption of petrol and the distance travelled by a car is a case of direct proportion. Similarly, the total amount spent and the number of articles purchased is also an example of direct proportion.

© NCERT not to be republished

Think of a few more examples for direct proportion. Check whether Mohan [in the initial example] will take 750 mL of water, 5 spoons of sugar, $2\frac{1}{2}$ spoons of tea leaves and 125 mL of milk to prepare tea for five persons! Let us try to understand further the concept of direct proportion through the following activities.

DO THIS

(i)
- Take a clock and fix its minute hand at 12.
- Record the angle turned through by the minute hand from its original position and the time that has passed, in the following table:

Time Passed (T) (in minutes)	(T_1) 15	(T_2) 30	(T_3) 45	(T_4) 60
Angle turned (A) (in degree)	(A_1) 90	(A_2) ...	(A_3) ...	(A_4) ...
$\frac{T}{A}$	...	...	...	...

What do you observe about T and A? Do they increase together?

Is $\frac{T}{A}$ same every time?

Is the angle turned through by the minute hand directly proportional to the time that has passed? Yes!

From the above table, you can also see

$$T_1 : T_2 = A_1 : A_2 \text{, because}$$
$$T_1 : T_2 = 15 : 30 \quad = 1:2$$
$$A_1 : A_2 = 90 : 180 \quad = 1:2$$

Check if $T_2 : T_3 = A_2 : A_3$ and $T_3 : T_4 = A_3 : A_4$

You can repeat this activity by choosing your own time interval.

(ii) Ask your friend to fill the following table and find the ratio of his age to the corresponding age of his mother.

	Age five years ago	**Present age**	**Age after five years**
Friend's age (F)			
Mother's age (M)			
$\frac{F}{M}$			

What do you observe?

Do F and M increase (or decrease) together? Is $\frac{F}{M}$ same every time? No!

You can repeat this activity with other friends and write down your observations.

© NCERT not to be republished

Thus, variables increasing (or decreasing) together need not always be in direct proportion. For example:

(i) physical changes in human beings occur with time but not necessarily in a predetermined ratio.

(ii) changes in weight and height among individuals are not in any known proportion and

(iii) there is no direct relationship or ratio between the height of a tree and the number of leaves growing on its branches. Think of some more similar examples.

TRY THESE

1. Observe the following tables and find if x and y are directly proportional.

(i)

x	20	17	14	11	8	5	2
y	40	34	28	22	16	10	4

(ii)

x	6	10	14	18	22	26	30
y	4	8	12	16	20	24	28

(iii)

x	5	8	12	15	18	20
y	15	24	36	60	72	100

2. Principal = ₹ 1000, Rate = 8% per annum. Fill in the following table and find which type of interest (simple or compound) changes in direct proportion with time period.

$\frac{P \times r \times t}{100}$

$P\left(1+\frac{r}{100}\right)^{t} - P$

Time period	1 year	2 years	3 years
Simple Interest (in ₹)			
Compound Interest (in ₹)			

THINK, DISCUSS AND WRITE

If we fix time period and the rate of interest, simple interest changes proportionally with principal. Would there be a similar relationship for compound interest? Why?

Let us consider some solved examples where we would use the concept of direct proportion.

Example 1: The cost of 5 metres of a particular quality of cloth is ₹ 210. Tabulate the cost of 2, 4, 10 and 13 metres of cloth of the same type.

Solution: Suppose the length of cloth is x metres and its cost, in ₹, is y.

x	2	4	5	10	13
y	y_2	y_3	210	y_4	y_5

© NCERT not to be republished

As the length of cloth increases, cost of the cloth also increases in the same ratio. It is a case of direct proportion.

We make use of the relation of type $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

(i) Here $x_1 = 5$, $y_1 = 210$ and $x_2 = 2$

Therefore, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ gives $\frac{5}{210} = \frac{2}{y_2}$ or $5y_2 = 2 \times 210$ or $y_2 = \frac{2 \times 210}{5} = 84$

(ii) If $x_3 = 4$, then $\frac{5}{210} = \frac{4}{y_3}$ or $5y_3 = 4 \times 210$ or $y_3 = \frac{4 \times 210}{5} = 168$

[Can we use $\frac{x_2}{y_2} = \frac{x_3}{y_3}$ here? Try!]

(iii) If $x_4 = 10$, then $\frac{5}{210} = \frac{10}{y_4}$ or $y_4 = \frac{10 \times 210}{5} = 420$

(iv) If $x_5 = 13$, then $\frac{5}{210} = \frac{13}{y_5}$ or $y_5 = \frac{13 \times 210}{5} = 546$

[Note that here we can also use $\frac{2}{84}$ or $\frac{4}{168}$ or $\frac{10}{420}$ in the place of $\frac{5}{210}$]

Example 2: An electric pole, 14 metres high, casts a shadow of 10 metres. Find the height of a tree that casts a shadow of 15 metres under similar conditions.

Solution: Let the height of the tree be x metres. We form a table as shown below:

height of the object (in metres)	14	x
length of the shadow (in metres)	10	15

Note that more the height of an object, the more would be the length of its shadow.

Hence, this is a case of direct proportion. That is, $\frac{x_1}{y_1} = \frac{x_2}{y_2}$

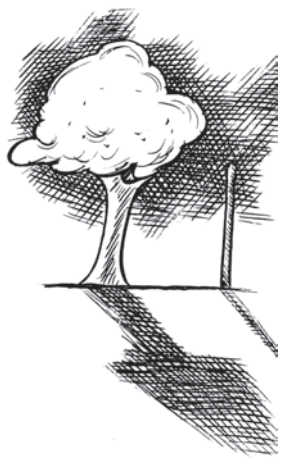
We have $\frac{14}{10} = \frac{x}{15}$ (Why?)

or $\frac{14}{10} \times 15 = x$

or $\frac{14 \times 3}{2} = x$

So $21 = x$

Thus, height of the tree is 21 metres.



Alternately, we can write $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ as $\frac{x_1}{x_2} = \frac{y_1}{y_2}$

© NCERT not to be republished

so $x_1 : x_2 = y_1 : y_2$

or $14 : x = 10 : 15$

Therefore, $10 \times x = 15 \times 14$

or $x = \dfrac{15 \times 14}{10} = 21$

Example 3: If the weight of 12 sheets of thick paper is 40 grams, how many sheets of the same paper would weigh $2\frac{1}{2}$ kilograms?

Solution:

Let the number of sheets which weigh $2\frac{1}{2}$ kg be x. We put the above information in the form of a table as shown below:

Number of sheets	12	x
Weight of sheets (in grams)	40	2500

1 kilogram = 1000 grams

$2\frac{1}{2}$ kilograms = 2500 grams

More the number of sheets, the more would their weight be. So, the number of sheets and their weights are directly proportional to each other.

So, $\dfrac{12}{40} = \dfrac{x}{2500}$

or $\dfrac{12 \times 2500}{40} = x$

or $750 = x$

Thus, the required number of sheets of paper = 750.

Alternate method:

Two quantities x and y which vary in direct proportion have the relation $x = ky$ or $\dfrac{x}{y} = k$

Here, $k = \dfrac{\text{number of sheets}}{\text{weight of sheets in grams}} = \dfrac{12}{40} = \dfrac{3}{10}$

Now x is the number of sheets of the paper which weigh $2\frac{1}{2}$ kg [2500 g].

Using the relation $x = ky$, $x = \dfrac{3}{10} \times 2500 = 750$

Thus, 750 sheets of paper would weigh $2\frac{1}{2}$ kg.

Example 4: A train is moving at a uniform speed of 75 km/hour.

(i) How far will it travel in 20 minutes?

(ii) Find the time required to cover a distance of 250 km.

Solution: Let the distance travelled (in km) in 20 minutes be x and time taken (in minutes) to cover 250 km be y.

1 hour = 60 minutes

Distance travelled (in km)	75	x	250
Time taken (in minutes)	60	20	y

© NCERT not to be republished

Since the speed is uniform, therefore, the distance covered would be directly proportional to time.

(i) We have $\frac{75}{60} = \frac{x}{20}$

or $\frac{75}{60} \times 20 = x$

or $x = 25$

So, the train will cover a distance of 25 km in 20 minutes.

(ii) Also, $\frac{75}{60} = \frac{250}{y}$

or $y = \frac{250 \times 60}{75} = 200$ minutes or 3 hours 20 minutes.

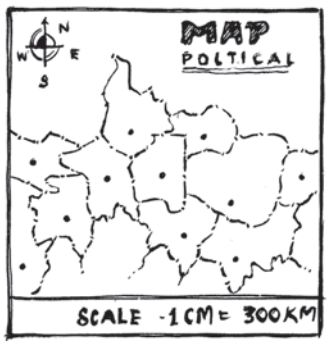
Therefore, 3 hours 20 minutes will be required to cover a distance of 250 kilometres.

Alternatively, when x is known, then one can determine y from the relation $\frac{x}{20} = \frac{250}{y}$.

You know that a map is a miniature representation of a very large region. A scale is usually given at the bottom of the map. The scale shows a relationship between actual length and the length represented on the map. The scale of the map is thus the ratio of the distance between two points on the map to the actual distance between two points on the large region.

For example, if 1 cm on the map represents 8 km of actual distance [i.e., the scale is 1 cm : 8 km or 1 : 800,000] then 2 cm on the same map will represent 16 km. Hence, we can say that scale of a map is based on the concept of direct proportion.

Example 5: The scale of a map is given as 1:30000000. Two cities are 4 cm apart on the map. Find the actual distance between them.



Solution: Let the map distance be x cm and actual distance be y cm, then

$$1:30000000 = x : y$$

or $$\frac{1}{3 \times 10^7} = \frac{x}{y}$$

Since $x = 4$ so, $$\frac{1}{3 \times 10^7} = \frac{4}{y}$$

or $$y = 4 \times 3 \times 10^7 = 12 \times 10^7 \text{ cm} = 1200 \text{ km}.$$

Thus, two cities, which are 4 cm apart on the map, are actually 1200 km away from each other.

DO THIS

Take a map of your State. Note the scale used there. Using a ruler, measure the "map distance" between any two cities. Calculate the actual distance between them.

© NCERT not to be republished

EXERCISE 11.1

1. Following are the car parking charges near a railway station upto

4 hours	₹ 60
8 hours	₹ 100
12 hours	₹ 140
24 hours	₹ 180

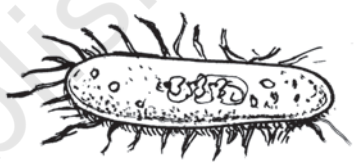
Check if the parking charges are in direct proportion to the parking time.

2. A mixture of paint is prepared by mixing 1 part of red pigments with 8 parts of base. In the following table, find the parts of base that need to be added.

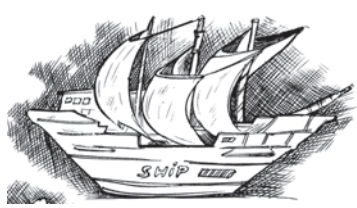
Parts of red pigment	1	4	7	12	20
Parts of base	8	...	...	...	...

3. In Question 2 above, if 1 part of a red pigment requires 75 mL of base, how much red pigment should we mix with 1800 mL of base?

4. A machine in a soft drink factory fills 840 bottles in six hours. How many bottles will it fill in five hours?



5. A photograph of a bacteria enlarged 50,000 times attains a length of 5 cm as shown in the diagram. What is the *actual* length of the bacteria? If the photograph is enlarged 20,000 times only, what would be its enlarged length?



6. In a model of a ship, the mast is 9 cm high, while the mast of the actual ship is 12 m high. If the length of the ship is 28 m, how long is the model ship?

7. Suppose 2 kg of sugar contains 9×10^6 crystals. How many sugar crystals are there in (i) 5 kg of sugar? (ii) 1.2 kg of sugar?

8. Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?

9. A 5 m 60 cm high vertical pole casts a shadow 3 m 20 cm long. Find at the same time (i) the length of the shadow cast by another pole 10 m 50 cm high (ii) the height of a pole which casts a shadow 5m long.

10. A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?

DO THIS

1. On a squared paper, draw five squares of different sides. Write the following information in a tabular form.

	Square-1	**Square-2**	**Square-3**	**Square-4**	**Square-5**
Length of a side (L)					
Perimeter (P)					
$\frac{L}{P}$					

© NCERT not to be republished

Area (A)					
$\frac{L}{A}$					

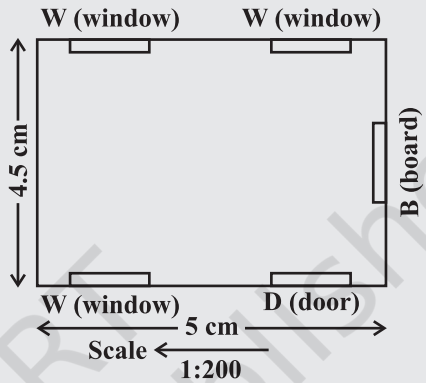
Find whether the length of a side is in direct proportion to:

(a) the perimeter of the square.

(b) the area of the square.

2. The following ingredients are required to make halwa for 5 persons:
Suji/Rawa = 250 g, Sugar = 300 g,
Ghee = 200 g, Water = 500 mL.
Using the concept of proportion, estimate the changes in the quantity of ingredients, to prepare halwa for your class.

3. Choose a scale and make a map of your classroom, showing windows, doors, blackboard etc. (An example is given here).



THINK, DISCUSS AND WRITE

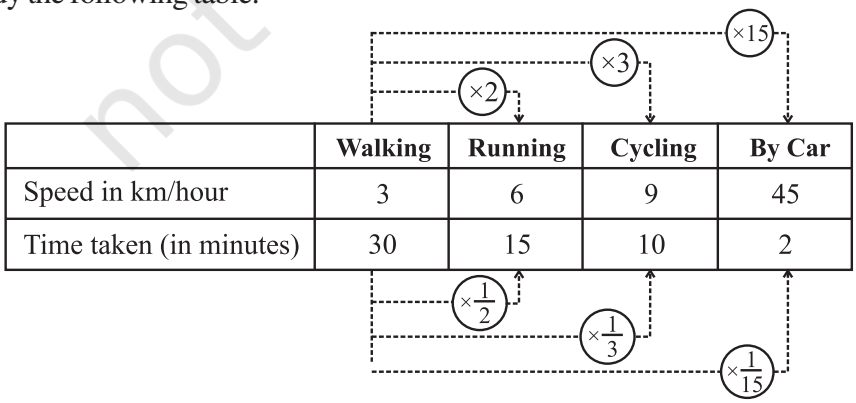
Take a few problems discussed so far under 'direct variation'. Do you think that they can be solved by 'unitary method'?

11.3 Inverse Proportion

Two quantities may change in such a manner that if one quantity increases, the other quantity decreases and vice versa. For example, as the number of workers increases, time taken to finish the job decreases. Similarly, if we increase the speed, the time taken to cover a given distance decreases.

To understand this, let us look into the following situation.

Zaheeda can go to her school in four different ways. She can walk, run, cycle or go by car. Study the following table.



© NCERT not to be republished

Observe that as the speed increases, time taken to cover the same distance decreases.

As Zaheeda doubles her speed by running, time reduces to half. As she increases her speed to three times by cycling, time decreases to one third. Similarly, as she increases her speed to 15 times, time decreases to one fifteenth. (Or, in other words the ratio by which time decreases is inverse of the ratio by which the corresponding speed increases). Can we say that speed and time change inversely in proportion?

> Multiplicative inverse of a number is its reciprocal. Thus, $\frac{1}{2}$ is the inverse of 2 and vice versa. (Note that $2 \times \frac{1}{2} = \frac{1}{2} \times 2 = 1$).

Let us consider another example. A school wants to spend ₹ 6000 on mathematics textbooks. How many books could be bought at ₹ 40 each? Clearly 150 books can be bought. If the price of a textbook is more than ₹ 40, then the number of books which could be purchased with the same amount of money would be less than 150. Observe the following table.

Price of each book (in ₹)	40	50	60	75	80	100
Number of books that can be bought	150	120	100	80	75	60

What do you observe? You will appreciate that as the price of the books increases, the number of books that can be bought, keeping the fund constant, will decrease.

Ratio by which the price of books increases when going from 40 to 50 is 4 : 5, and the ratio by which the corresponding number of books decreases from 150 to 120 is 5 : 4. This means that the two ratios are inverses of each other.

Notice that the product of the corresponding values of the two quantities is constant; that is, $40 \times 150 = 50 \times 120 = 6000$.

If we represent the price of one book as x and the number of books bought as y, then as x increases y decreases and vice-versa. It is important to note that the product xy remains constant. We say that x varies inversely with y and y varies inversely with x. *Thus two quantities x and y are said to vary in inverse proportion, if there exists a relation of the type xy = k between them, k being a constant. If* y_1, y_2 *are the values of y corresponding to the values* x_1, x_2 *of x respectively then* $x_1y_1 = x_2y_2 (= k)$, *or* $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

We say that x and y are in **inverse proportion**.

Hence, in this example, cost of a book and number of books purchased in a fixed amount are inversely proportional. Similarly, speed of a vehicle and the time taken to cover a fixed distance changes in inverse proportion.

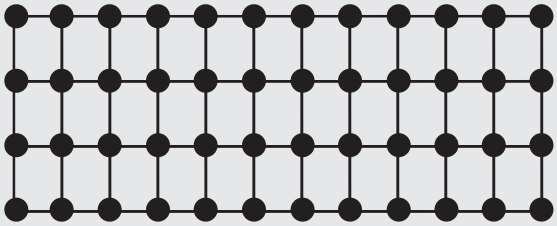
Think of more such examples of pairs of quantities that vary in inverse proportion. You may now have a look at the furniture – arranging problem, stated in the introductory part of this chapter.

Here is an activity for better understanding of the inverse proportion.

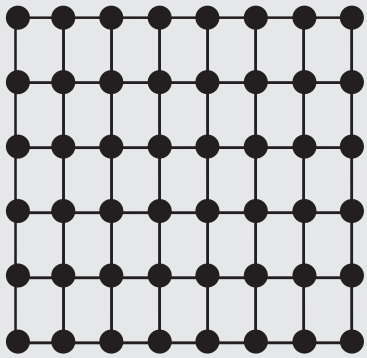
© NCERT not to be republished

DO THIS

Take a squared paper and arrange 48 counters on it in different number of rows as shown below.



4 Rows, 12 columns



6 Rows, 8 columns

Number of Rows (R)	(R_1) 2	(R_2) 3	(R_3) 4	(R_4) 6	(R_5) 8
Number of Columns (C)	(C_1) ...	(C_2) ...	(C_3) 12	(C_4) 8	(C_5) ...

What do you observe? As R increases, C decreases.

(i) Is $R_1 : R_2 = C_2 : C_1$? (ii) Is $R_3 : R_4 = C_4 : C_3$?

(iii) Are R and C inversely proportional to each other?

Try this activity with 36 counters.

TRY THESE

Observe the following tables and find which pair of variables (here x and y) are in inverse proportion.

(i)

x	50	40	30	20
y	5	6	7	8

(ii)

x	100	200	300	400
y	60	30	20	15

(iii)

x	90	60	45	30	20	5
y	10	15	20	25	30	35

Let us consider some examples where we use the concept of inverse proportion.

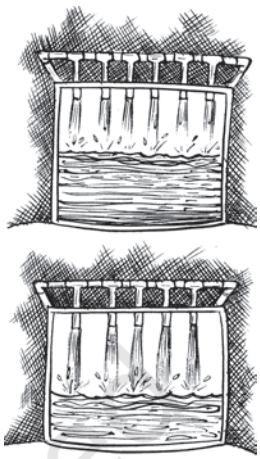
When two quantities x and y are in direct proportion (or vary directly) they are also written as $x \propto y$.

When two quantities x and y are in inverse proportion (or vary inversely) they are also written as $x \propto \frac{1}{y}$.

© NCERT not to be republished

Example 7: 6 pipes are required to fill a tank in 1 hour 20 minutes. How long will it take if only 5 pipes of the same type are used?

Solution:



Let the desired time to fill the tank be x minutes. Thus, we have the following table.

Number of pipes	6	5
Time (in minutes)	80	x

Lesser the number of pipes, more will be the time required by it to fill the tank. So, this is a case of inverse proportion.

Hence, $80 \times 6 = x \times 5$ $[x_1 y_1 = x_2 y_2]$

or $\frac{80 \times 6}{5} = x$

or $x = 96$

Thus, time taken to fill the tank by 5 pipes is 96 minutes or 1 hour 36 minutes.

Example 8: There are 100 students in a hostel. Food provision for them is for 20 days. How long will these provisions last, if 25 more students join the group?

Solution: Suppose the provisions last for y days when the number of students is 125. We have the following table.

Number of students	100	125
Number of days	20	y

Note that more the number of students, the sooner would the provisions exhaust. Therefore, this is a case of inverse proportion.

So, $100 \times 20 = 125 \times y$

or $\frac{100 \times 20}{125} = y$ or $16 = y$

Thus, the provisions will last for 16 days, if 25 more students join the hostel.

Alternately, we can write $x_1 y_1 = x_2 y_2$ as $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

That is, $x_1 : x_2 = y_2 : y_1$

or $100 : 125 = y : 20$

or $y = \frac{100 \times 20}{125} = 16$

Example 9: If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?

Solution:

Let the number of workers employed to build the wall in 30 hours be y.

© NCERT not to be republished

We have the following table.

Number of hours	48	30
Number of workers	15	y

Obviously more the number of workers, faster will they build the wall.
So, the number of hours and number of workers vary in inverse proportion.

So $48 \times 15 = 30 \times y$

Therefore, $\frac{48 \times 15}{30} = y$ or $y = 24$

i.e., to finish the work in 30 hours, 24 workers are required.

EXERCISE 11.2

1. Which of the following are in inverse proportion?

(i) The number of workers on a job and the time to complete the job.

(ii) The time taken for a journey and the distance travelled in a uniform speed.

(iii) Area of cultivated land and the crop harvested.

(iv) The time taken for a fixed journey and the speed of the vehicle.

(v) The population of a country and the area of land per person.

2. In a Television game show, the prize money of ₹ 1,00,000 is to be divided equally amongst the winners. Complete the following table and find whether the prize money given to an individual winner is directly or inversely proportional to the number of winners?

Number of winners	1	2	4	5	8	10	20
Prize for each winner (in ₹)	1,00,000	50,000	...	...	...	...	...

3. Rehman is making a wheel using spokes. He wants to fix equal spokes in such a way that the angles between any pair of consecutive spokes are equal. Help him by completing the following table.

Number of spokes	4	6	8	10	12
Angle between a pair of consecutive spokes	90°	60°	...	...	...

© NCERT not to be republished

(i) Are the number of spokes and the angles formed between the pairs of consecutive spokes in inverse proportion?

(ii) Calculate the angle between a pair of consecutive spokes on a wheel with 15 spokes.

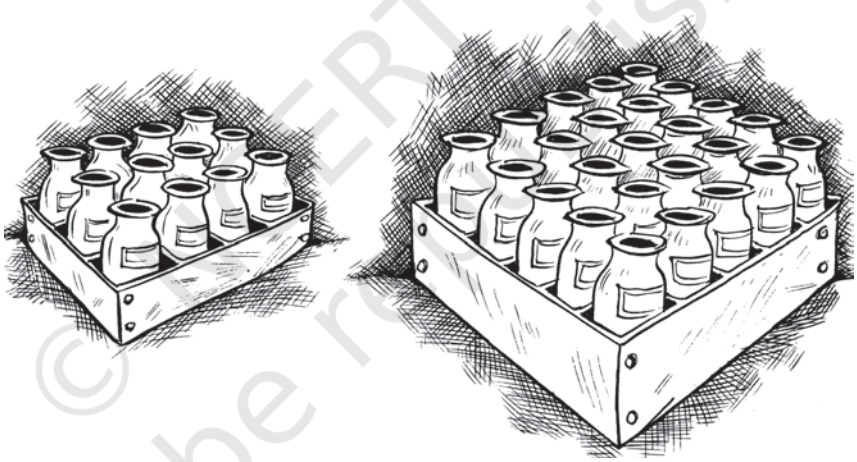
(iii) How many spokes would be needed, if the angle between a pair of consecutive spokes is 40°?

4. If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

5. A farmer has enough food to feed 20 animals in his cattle for 6 days. How long would the food last if there were 10 more animals in his cattle?

6. A contractor estimates that 3 persons could rewire Jasminder's house in 4 days. If, he uses 4 persons instead of three, how long should they take to complete the job?

7. A batch of bottles were packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?



8. A factory requires 42 machines to produce a given number of articles in 63 days. How many machines would be required to produce the same number of articles in 54 days?

9. A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h?

10. Two persons could fit new windows in a house in 3 days.

(i) One of the persons fell ill before the work started. How long would the job take now?

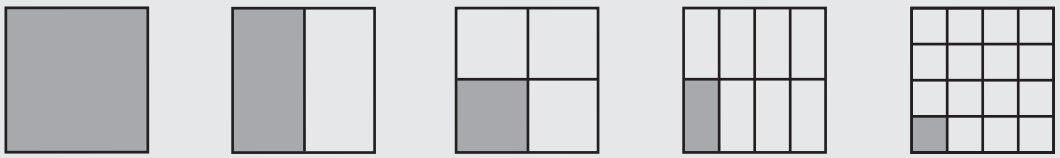
(ii) How many persons would be needed to fit the windows in one day?

11. A school has 8 periods a day each of 45 minutes duration. How long would each period be, if the school has 9 periods a day, assuming the number of school hours to be the same?

© NCERT not to be republished

DO THIS

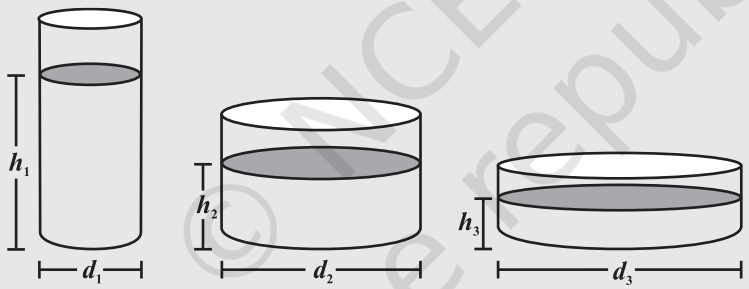
1. Take a sheet of paper. Fold it as shown in the figure. Count the number of parts and the area of a part in each case.



Tabulate your observations and discuss with your friends. Is it a case of inverse proportion? Why?

Number of parts	1	2	4	8	16
Area of each part	area of the paper	$\frac{1}{2}$ the area of the paper	...	...	...

2. Take a few containers of different sizes with circular bases. Fill the same amount of water in each container. Note the diameter of each container and the respective height at which the water level stands. Tabulate your observations. Is it a case of inverse proportion?



Diameter of container (in cm)			
Height of water level (in cm)			

WHAT HAVE WE DISCUSSED?

1. Two quantities x and y are said to be in **direct proportion** if they increase (decrease) together in such a manner that the ratio of their corresponding values remains constant. That is if $\frac{x}{y} = k$ [k is a positive number], then x and y are said to vary directly. In such a case if y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$.

© NCERT not to be republished

2. Two quantities x and y are said to be in **inverse proportion** if an increase in x causes a proportional decrease in y (and vice-versa) in such a manner that the product of their corresponding values remains constant. That is, if $xy = k$, then x and y are said to vary inversely. In this case if y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $x_1 y_1 = x_2 y_2$ or $\frac{x_1}{x_2} = \frac{y_2}{y_1}$.

© NCERT not to be republished

CHAPTER

12

Factorisation

0852CH14

12.1 Introduction

12.1.1 Factors of natural numbers

You will remember what you learnt about factors in Class VI. Let us take a natural number, say 30, and write it as a product of other natural numbers, say

$$30 = 2 \times 15$$
$$= 3 \times 10 = 5 \times 6$$

Thus, 1, 2, 3, 5, 6, 10, 15 and 30 are the factors of 30. Of these, 2, 3 and 5 are the prime factors of 30 (Why?)

A number written as a product of prime factors is said to be in the prime factor form; for example, 30 written as $2 \times 3 \times 5$ is in the prime factor form.

The prime factor form of 70 is $2 \times 5 \times 7$.

The prime factor form of 90 is $2 \times 3 \times 3 \times 5$, and so on.

Similarly, we can express algebraic expressions as products of their factors. This is what we shall learn to do in this chapter.

> We know that 30 can also be written as
> $30 = 1 \times 30$
> Thus, 1 and 30 are also factors of 30. You will notice that 1 is a factor of any number. For example, $101 = 1 \times 101$. However, when we write a number as a product of factors, we shall not write 1 as a factor, unless it is specially required.

12.1.2 Factors of algebraic expressions

We have seen in Class VII that in algebraic expressions, terms are formed as products of factors. For example, in the algebraic expression $5xy + 3x$ the term $5xy$ has been formed by the factors 5, x and y, i.e.,

$$5xy = 5 \times x \times y$$

Observe that the factors 5, x and y of $5xy$ cannot further be expressed as a product of factors. We may say that 5, x and y are 'prime' factors of $5xy$. In algebraic expressions, we use the word 'irreducible' in place of 'prime'. We say that $5 \times x \times y$ is the irreducible form of $5xy$. Note $5 \times (xy)$ is not an irreducible form of $5xy$, since the factor xy can be further expressed as a product of x and y, i.e., $xy = x \times y$.

> Note 1 is a factor of $5xy$, since
> $5xy = 1 \times 5 \times x \times y$
> In fact, 1 is a factor of every term. As in the case of natural numbers, unless it is specially required, we do not show 1 as a separate factor of any term.

© NCERT not to be republished

Next consider the expression $3x\,(x+2)$. It can be written as a product of factors. 3, x and $(x+2)$

$$3x(x+2) = 3\times x\times (x+2)$$

The factors 3, x and $(x+2)$ are irreducible factors of $3x\,(x+2)$.

Similarly, the expression $10x\,(x+2)\,(y+3)$ is expressed in its irreducible factor form as $10x\,(x+2)\,(y+3) = 2\times 5\times x\times (x+2)\times (y+3)$.

12.2 What is Factorisation?

When we factorise an algebraic expression, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions.

Expressions like $3xy$, $5x^2y$, $2x\,(y+2)$, $5\,(y+1)\,(x+2)$ are already in factor form. Their factors can be just read off from them, as we already know.

On the other hand consider expressions like $2x+4$, $3x+3y$, x^2+5x, x^2+5x+6. It is not obvious what their factors are. We need to develop systematic methods to factorise these expressions, i.e., to find their factors. This is what we shall do now.

12.2.1 Method of common factors

- We begin with a simple example: Factorise $2x+4$.
  We shall write each term as a product of irreducible factors;

$$2x = 2\times x$$
$$4 = 2\times 2$$

Hence $$2x+4 = (2\times x)+(2\times 2)$$

Notice that factor 2 is common to both the terms.

Observe, by distributive law

$$2\times (x+2) = (2\times x)+(2\times 2)$$

Therefore, we can write

$$2x+4 = 2\times (x+2) = 2\,(x+2)$$

Thus, the expression $2x+4$ is the same as $2\,(x+2)$. Now we can read off its factors: they are 2 and $(x+2)$. These factors are irreducible.

Next, factorise $5xy+10x$.

The irreducible factor forms of $5xy$ and $10x$ are respectively,

$$5xy = 5\times x\times y$$
$$10x = 2\times 5\times x$$

Observe that the two terms have 5 and x as common factors. Now,

$$5xy+10x = (5\times x\times y)+(5\times x\times 2)$$
$$= (5x\times y)+(5x\times 2)$$

We combine the two terms using the distributive law,

$$(5x\times y)+(5x\times 2) = 5x\times (y+2)$$

Therefore, $5xy+10x = 5x\,(y+2)$. (This is the desired factor form.)

© NCERT not to be republished

Example 1: Factorise $12a^2b + 15ab^2$

Solution: We have $12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

$15ab^2 = 3 \times 5 \times a \times b \times b$

The two terms have 3, a and b as common factors.

Therefore, $12a^2b + 15ab^2 = (3 \times a \times b \times 2 \times 2 \times a) + (3 \times a \times b \times 5 \times b)$

$= 3 \times a \times b \times [(2 \times 2 \times a) + (5 \times b)]$ (combining the terms)

$= 3ab \times (4a + 5b)$

$= 3ab\ (4a + 5b)$ (required factor form)

Example 2: Factorise $10x^2 - 18x^3 + 14x^4$

Solution: $10x^2 = 2 \times 5 \times x \times x$

$18x^3 = 2 \times 3 \times 3 \times x \times x \times x$

$14x^4 = 2 \times 7 \times x \times x \times x \times x$

The common factors of the three terms are 2, x and x.

Therefore, $10x^2 - 18x^3 + 14x^4 = (2 \times x \times x \times 5) - (2 \times x \times x \times 3 \times 3 \times x)$

$+ (2 \times x \times x \times 7 \times x \times x)$

$= 2 \times x \times x \times [(5 - (3 \times 3 \times x) + (7 \times x \times x)]$ (combining the three terms)

$= 2x^2 \times (5 - 9x + 7x^2) = 2x^2(7x^2 - 9x + 5)$

TRY THESE

Factorise: (i) $12x + 36$ (ii) $22y - 33z$ (iii) $14pq + 35pqr$

Do you notice that the factor form of an expression has only one term?

12.2.2 Factorisation by regrouping terms

Look at the expression $2xy + 2y + 3x + 3$. You will notice that the first two terms have common factors 2 and y and the last two terms have a common factor 3. But there is no single factor common to all the terms. How shall we proceed?

Let us write $(2xy + 2y)$ in the factor form:

$2xy + 2y = (2 \times x \times y) + (2 \times y)$

$= (2 \times y \times x) + (2 \times y \times 1)$

$= (2y \times x) + (2y \times 1) = 2y\ (x + 1)$

Similarly, $3x + 3 = (3 \times x) + (3 \times 1)$

$= 3 \times (x + 1) = 3\ (x + 1)$

Note, we need to show1 as a factor here. Why?

Hence, $2xy + 2y + 3x + 3 = 2y\ (x + 1) + 3\ (x + 1)$

Observe, now we have a common factor $(x + 1)$ in both the terms on the right hand side. Combining the two terms,

$2xy + 2y + 3x + 3 = 2y\ (x + 1) + 3\ (x + 1) = (x + 1)\ (2y + 3)$

The expression $2xy + 2y + 3x + 3$ is now in the form of a product of factors. Its factors are $(x + 1)$ and $(2y + 3)$. Note, these factors are irreducible.

© NCERT not to be republished

What is regrouping?

Suppose, the above expression was given as $2xy + 3 + 2y + 3x$; then it will not be easy to see the factorisation. Rearranging the expression, as $2xy + 2y + 3x + 3$, allows us to form groups $(2xy + 2y)$ and $(3x + 3)$ leading to factorisation. This is regrouping.

Regrouping may be possible in more than one ways. Suppose, we regroup the expression as: $2xy + 3x + 2y + 3$. This will also lead to factors. Let us try:

$$\begin{aligned} 2xy + 3x + 2y + 3 &= 2 \times x \times y + 3 \times x + 2 \times y + 3 \\ &= x \times (2y + 3) + 1 \times (2y + 3) \\ &= (2y + 3)\,(x + 1) \end{aligned}$$

The factors are the same (as they have to be), although they appear in different order.

Example 3: Factorise $6xy - 4y + 6 - 9x$.

Solution:

Step 1 Check if there is a common factor among all terms. There is none.

Step 2 Think of grouping. Notice that first two terms have a common factor $2y$;

$$6xy - 4y = 2y\,(3x - 2) \qquad \text{(a)}$$

What about the last two terms? Observe them. If you change their order to $-9x + 6$, the factor $(3x - 2)$ will come out;

$$\begin{aligned} -9x + 6 &= -3\,(3x) + 3\,(2) \\ &= -3\,(3x - 2) \end{aligned} \qquad \text{(b)}$$

Step 3 Putting (a) and (b) together,

$$\begin{aligned} 6xy - 4y + 6 - 9x &= 6xy - 4y - 9x + 6 \\ &= 2y\,(3x - 2) - 3\,(3x - 2) \\ &= (3x - 2)\,(2y - 3) \end{aligned}$$

The factors of $(6xy - 4y + 6 - 9x)$ are $(3x - 2)$ and $(2y - 3)$.

EXERCISE 12.1

1. Find the common factors of the given terms.

(i) $12x,\ 36$ (ii) $2y,\ 22xy$ (iii) $14\,pq,\ 28p^2q^2$

(iv) $2x,\ 3x^2,\ 4$ (v) $6\,abc,\ 24ab^2,\ 12\,a^2b$

(vi) $16\,x^3,\ -4x^2,\ 32x$ (vii) $10\,pq,\ 20qr,\ 30rp$

(viii) $3x^2\,y^3,\ 10x^3\,y^2,\ 6\,x^2\,y^2z$

2. Factorise the following expressions.

(i) $7x - 42$ (ii) $6p - 12q$ (iii) $7a^2 + 14a$

(iv) $-16\,z + 20\,z^3$ (v) $20\,l^2\,m + 30\,a\,l\,m$

(vi) $5\,x^2\,y - 15\,xy^2$ (vii) $10\,a^2 - 15\,b^2 + 20\,c^2$

(viii) $-4\,a^2 + 4\,ab - 4\,ca$ (ix) $x^2\,y\,z + x\,y^2z + x\,y\,z^2$

(x) $a\,x^2\,y + b\,x\,y^2 + c\,x\,y\,z$

3. Factorise.

(i) $x^2 + x\,y + 8x + 8y$ (ii) $15\,xy - 6x + 5y - 2$

© NCERT not to be republished

(iii) $ax + bx - ay - by$ (iv) $15\,pq + 15 + 9q + 25p$

(v) $z - 7 + 7\,x\,y - x\,y\,z$

12.2.3 Factorisation using identities

We know that

$$(a + b)^2 = a^2 + 2ab + b^2 \quad \text{(I)}$$

$$(a - b)^2 = a^2 - 2ab + b^2 \quad \text{(II)}$$

$$(a + b)(a - b) = a^2 - b^2 \quad \text{(III)}$$

The following solved examples illustrate how to use these identities for factorisation. What we do is to observe the given expression. If it has a form that fits the right hand side of one of the identities, then the expression corresponding to the left hand side of the identity gives the desired factorisation.

Example 4: Factorise $x^2 + 8x + 16$

Solution: Observe the expression; it has three terms. Therefore, it does not fit Identity III. Also, it's first and third terms are perfect squares with a positive sign before the middle term. So, it is of the form $a^2 + 2ab + b^2$ where $a = x$ and $b = 4$

such that $a^2 + 2ab + b^2 = x^2 + 2\,(x)\,(4) + 4^2$

$= x^2 + 8x + 16$

Since $a^2 + 2ab + b^2 = (a + b)^2$,

by comparison $x^2 + 8x + 16 = (x + 4)^2$ (the required factorisation)

Observe here the given expression is of the form $a^2 - 2ab + b^2$. Where $a = 2y$, and $b = 3$ with $2ab = 2 \times 2y \times 3 = 12y$.

Example 5: Factorise $4y^2 - 12y + 9$

Solution: Observe $4y^2 = (2y)^2$, $9 = 3^2$ and $12y = 2 \times 3 \times (2y)$

Therefore, $4y^2 - 12y + 9 = (2y)^2 - 2 \times 3 \times (2y) + (3)^2$

$= (2y - 3)^2$ (required factorisation)

Example 6: Factorise $49p^2 - 36$

Solution: There are two terms; both are squares and the second is negative. The expression is of the form $(a^2 - b^2)$. Identity III is applicable here;

$49p^2 - 36 = (7p)^2 - (6)^2$

$= (7p - 6)(7p + 6)$ (required factorisation)

Example 7: Factorise $a^2 - 2ab + b^2 - c^2$

Solution: The first three terms of the given expression form $(a - b)^2$. The fourth term is a square. So the expression can be reduced to a difference of two squares.

Thus, $a^2 - 2ab + b^2 - c^2 = (a - b)^2 - c^2$ (Applying Identity II)

$= [(a - b) - c)\,((a - b) + c)]$ (Applying Identity III)

$= (a - b - c)\,(a - b + c)$ (required factorisation)

Notice, how we applied two identities one after the other to obtain the required factorisation.

Example 8: Factorise $m^4 - 256$

Solution: We note $m^4 = (m^2)^2$ and $256 = (16)^2$

© NCERT not to be republished

Thus, the given expression fits Identity III.

Therefore, $m^4 - 256 = (m^2)^2 - (16)^2$

$= (m^2 - 16)(m^2 + 16)$ [(using Identity (III)]

Now, $(m^2 + 16)$ cannot be factorised further, but $(m^2 - 16)$ is factorisable again as per Identity III.

$m^2 - 16 = m^2 - 4^2$

$= (m - 4)(m + 4)$

Therefore, $m^4 - 256 = (m - 4)(m + 4)(m^2 + 16)$

12.2.4 Factors of the form $(x + a)(x + b)$

Let us now discuss how we can factorise expressions in one variable, like $x^2 + 5x + 6$, $y^2 - 7y + 12$, $z^2 - 4z - 12$, $3m^2 + 9m + 6$, etc. Observe that these expressions are not of the type $(a + b)^2$ or $(a - b)^2$, i.e., they are not perfect squares. For example, in $x^2 + 5x + 6$, the term 6 is not a perfect square. These expressions obviously also do not fit the type $(a^2 - b^2)$ either.

They, however, seem to be of the type $x^2 + (a + b)x + ab$. We may therefore, try to use Identity IV studied in the last chapter to factorise these expressions:

$$(x + a)(x + b) = x^2 + (a + b)x + ab \qquad \text{(IV)}$$

For that we have to look at the coefficients of x and the constant term. Let us see how it is done in the following example.

Example 9: Factorise $x^2 + 5x + 6$

Solution: If we compare the R.H.S. of Identity (IV) with $x^2 + 5x + 6$, we find $ab = 6$, and $a + b = 5$. From this, we must obtain a and b. The factors then will be $(x + a)$ and $(x + b)$.

If $ab = 6$, it means that a and b are factors of 6. Let us try $a = 6$, $b = 1$. For these values $a + b = 7$, and not 5, So this choice is not right.

Let us try $a = 2$, $b = 3$. For this $a + b = 5$ exactly as required.

The factorised form of this given expression is then $(x + 2)(x + 3)$.

In general, for factorising an algebraic expression of the type $x^2 + px + q$, we find two factors a and b of q (i.e., the constant term) such that

$ab = q$ and $a + b = p$

Then, the expression becomes	$x^2 + (a + b)x + ab$	
or	$x^2 + ax + bx + ab$	
or	$x(x + a) + b(x + a)$	
or	$(x + a)(x + b)$	which are the required factors.

Example 10: Find the factors of $y^2 - 7y + 12$.

Solution: We note $12 = 3 \times 4$ and $3 + 4 = 7$. Therefore,

$y^2 - 7y + 12 = y^2 - 3y - 4y + 12$

$= y(y - 3) - 4(y - 3) = (y - 3)(y - 4)$

© NCERT not to be republished

Note, this time we did not compare the expression with that in Identity (IV) to identify a and b. After sufficient practice you may not need to compare the given expressions for their factorisation with the expressions in the identities; instead you can proceed directly as we did above.

Example 11: Obtain the factors of $z^2 - 4z - 12$.

Solution: Here $a\,b = -12$; this means one of a and b is negative. Further, $a + b = -4$, this means the one with larger numerical value is negative. We try $a = -4$, $b = 3$; but this will not work, since $a + b = -1$. Next possible values are $a = -6$, $b = 2$, so that $a + b = -4$ as required.

Hence,
$$z^2 - 4z - 12 = z^2 - 6z + 2z - 12$$
$$= z(z - 6) + 2(z - 6)$$
$$= (z - 6)(z + 2)$$

Example 12: Find the factors of $3m^2 + 9m + 6$.

Solution: We notice that 3 is a common factor of all the terms.

Therefore, $3m^2 + 9m + 6 = 3(m^2 + 3m + 2)$

Now, $m^2 + 3m + 2 = m^2 + m + 2m + 2$ (as $2 = 1 \times 2$)
$$= m(m + 1) + 2(m + 1)$$
$$= (m + 1)(m + 2)$$

Therefore, $3m^2 + 9m + 6 = 3(m + 1)(m + 2)$

EXERCISE 12.2

1. Factorise the following expressions.

(i) $a^2 + 8a + 16$ (ii) $p^2 - 10p + 25$ (iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$ (v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$ (Hint: Expand $(l + m)^2$ first)

(viii) $a^4 + 2a^2b^2 + b^4$

2. Factorise.

(i) $4p^2 - 9q^2$ (ii) $63a^2 - 112b^2$ (iii) $49x^2 - 36$

(iv) $16x^5 - 144x^3$ (v) $(l + m)^2 - (l - m)^2$

(vi) $9x^2y^2 - 16$ (vii) $(x^2 - 2xy + y^2) - z^2$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

3. Factorise the expressions.

(i) $ax^2 + bx$ (ii) $7p^2 + 21q^2$ (iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$ (v) $(lm + l) + m + 1$

(vi) $y(y + z) + 9(y + z)$ (vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$ (ix) $6xy - 4y + 6 - 9x$

© NCERT not to be republished

4. Factorise.

(i) $a^4 - b^4$ (ii) $p^4 - 81$ (iii) $x^4 - (y + z)^4$

(iv) $x^4 - (x - z)^4$ (v) $a^4 - 2a^2b^2 + b^4$

5. Factorise the following expressions.

(i) $p^2 + 6p + 8$ (ii) $q^2 - 10q + 21$ (iii) $p^2 + 6p - 16$

12.3 Division of Algebraic Expressions

We have learnt how to add and subtract algebraic expressions. We also know how to multiply two expressions. We have not however, looked at division of one algebraic expression by another. This is what we wish to do in this section.

We recall that division is the inverse operation of multiplication. Thus, $7 \times 8 = 56$ gives $56 \div 8 = 7$ or $56 \div 7 = 8$.

We may similarly follow the division of algebraic expressions. For example,

(i) $2x \times 3x^2 = 6x^3$

Therefore, $6x^3 \div 2x = 3x^2$

and also, $6x^3 \div 3x^2 = 2x$.

(ii) $5x\,(x + 4) = 5x^2 + 20x$

Therefore, $(5x^2 + 20x) \div 5x = x + 4$

and also $(5x^2 + 20x) \div (x + 4) = 5x$.

We shall now look closely at how the division of one expression by another can be carried out. To begin with we shall consider the division of a monomial by another monomial.

12.3.1 Division of a monomial by another monomial

Consider $6x^3 \div 2x$

We may write $2x$ and $6x^3$ in irreducible factor forms,

$$2x = 2 \times x$$

$$6x^3 = 2 \times 3 \times x \times x \times x$$

Now we group factors of $6x^3$ to separate $2x$,

$$6x^3 = 2 \times x \times (3 \times x \times x) = (2x) \times (3x^2)$$

Therefore, $6x^3 \div 2x = 3x^2$.

A shorter way to depict cancellation of common factors is as we do in division of numbers:

$$77 \div 7 = \frac{77}{7} = \frac{7 \times 11}{7} = 11$$

Similarly, $$6x^3 \div 2x = \frac{6x^3}{2x}$$

$$= \frac{2 \times 3 \times x \times x \times x}{2 \times x} = 3 \times x \times x = 3x^2$$

Example 13: Do the following divisions.

(i) $-20x^4 \div 10x^2$ (ii) $7x^2y^2z^2 \div 14xyz$

Solution:

(i) $-20x^4 = -2 \times 2 \times 5 \times x \times x \times x \times x$

$10x^2 = 2 \times 5 \times x \times x$

© NCERT not to be republished

Therefore, $(-20x^4) \div 10x^2 = \dfrac{-2\times 2\times 5\times x\times x\times x\times x}{2\times 5\times x\times x} = -2\times x\times x = -2x^2$

(ii) $7x^2y^2z^2 \div 14xyz = \dfrac{7\times x\times x\times y\times y\times z\times z}{2\times 7\times x\times y\times z}$

$= \dfrac{x\times y\times z}{2} = \dfrac{1}{2}xyz$

TRY THESE

Divide.

(i) $24xy^2z^3$ by $6yz^2$ (ii) $63a^2b^4c^6$ by $7a^2b^2c^3$

12.3.2 Division of a polynomial by a monomial

Let us consider the division of the trinomial $4y^3 + 5y^2 + 6y$ by the monomial $2y$.

$4y^3 + 5y^2 + 6y = (2\times 2\times y\times y\times y) + (5\times y\times y) + (2\times 3\times y)$

(Here, we expressed each term of the polynomial in factor form) we find that $2\times y$ is common in each term. Therefore, separating $2\times y$ from each term. We get

$4y^3 + 5y^2 + 6y = 2\times y\times (2\times y\times y) + 2\times y\times \left(\dfrac{5}{2}\times y\right) + 2\times y\times 3$

$= 2y\,(2y^2) + 2y\left(\dfrac{5}{2}y\right) + 2y\,(3)$

$= 2y\left(2y^2 + \dfrac{5}{2}y + 3\right)$ (The common factor $2y$ is shown separately.

Therefore, $(4y^3 + 5y^2 + 6y) \div 2y$

$= \dfrac{4y^3+5y^2+6y}{2y} = \dfrac{2y\left(2y^2+\frac{5}{2}y+3\right)}{2y} = 2y^2 + \dfrac{5}{2}y + 3$

Here, we divide each term of the polynomial in the numerator by the monomial in the denominator.

Alternatively, we could divide each term of the trinomial by the monomial using the cancellation method.

$(4y^3 + 5y^2 + 6y) \div 2y = \dfrac{4y^3 + 5y^2 + 6y}{2y}$

$= \dfrac{4y^3}{2y} + \dfrac{5y^2}{2y} + \dfrac{6y}{2y} = 2y^2 + \dfrac{5}{2}y + 3$

Example 14: Divide $24(x^2yz + xy^2z + xyz^2)$ by $8xyz$ using both the methods.

Solution: $24\,(x^2yz + xy^2z + xyz^2)$

$= 2\times 2\times 2\times 3\times [(x\times x\times y\times z) + (x\times y\times y\times z) + (x\times y\times z\times z)]$

$= 2\times 2\times 2\times 3\times x\times y\times z\times (x + y + z) = 8\times 3\times xyz\times (x + y + z)$ (By taking out the common factor)

Therefore, $24\,(x^2yz + xy^2z + xyz^2) \div 8xyz$

$= \dfrac{8\times 3\times xyz\times (x + y + z)}{8\times xyz} = 3\times (x + y + z) = 3\,(x + y + z)$

© NCERT not to be republished

Alternately, $24(x^2yz + xy^2z + xyz^2) \div 8xyz = \frac{24x^2yz}{8xyz} + \frac{24xy^2z}{8xyz} + \frac{24xyz^2}{8xyz}$

$= 3x + 3y + 3z = 3(x + y + z)$

12.4 Division of Algebraic Expressions Continued (Polynomial ÷ Polynomial)

- Consider $(7x^2 + 14x) \div (x + 2)$

We shall factorise $(7x^2 + 14x)$ first to check and match factors with the denominator:

$7x^2 + 14x = (7 \times x \times x) + (2 \times 7 \times x)$

$= 7 \times x \times (x + 2) = 7x(x + 2)$

Will it help here to divide each term of the numerator by the binomial in the denominator?

Now $(7x^2 + 14x) \div (x + 2) = \frac{7x^2 + 14x}{x + 2}$

$= \frac{7x(x + 2)}{x + 2} = 7x$ (Cancelling the factor $(x + 2)$)

Example 15: Divide $44(x^4 - 5x^3 - 24x^2)$ by $11x(x - 8)$

Solution: Factorising $44(x^4 - 5x^3 - 24x^2)$, we get

$44(x^4 - 5x^3 - 24x^2) = 2 \times 2 \times 11 \times x^2(x^2 - 5x - 24)$

(taking the common factor x^2 out of the bracket)

$= 2 \times 2 \times 11 \times x^2(x^2 - 8x + 3x - 24)$

$= 2 \times 2 \times 11 \times x^2 [x(x - 8) + 3(x - 8)]$

$= 2 \times 2 \times 11 \times x^2 (x + 3)(x - 8)$

Therefore, $44(x^4 - 5x^3 - 24x^2) \div 11x(x - 8)$

$= \frac{2 \times 2 \times 11 \times x \times x \times (x + 3) \times (x - 8)}{11 \times x \times (x - 8)}$

$= 2 \times 2 \times x (x + 3) = 4x(x + 3)$

We cancel the factors 11, x and $(x - 8)$ common to both the numerator and denominator.

Example 16: Divide $z(5z^2 - 80)$ by $5z(z + 4)$

Solution: Dividend $= z(5z^2 - 80)$

$= z[(5 \times z^2) - (5 \times 16)]$

$= z \times 5 \times (z^2 - 16)$

$= 5z \times (z + 4)(z - 4)$ [using the identity $a^2 - b^2 = (a + b)(a - b)$]

Thus, $z(5z^2 - 80) \div 5z(z + 4) = \frac{5z(z - 4)(z + 4)}{5z(z + 4)} = (z - 4)$

© NCERT not to be republished

EXERCISE 12.3

1. Carry out the following divisions.

(i) $28x^4 \div 56x$ (ii) $-36y^3 \div 9y^2$ (iii) $66pq^2r^3 \div 11qr^2$

(iv) $34x^3y^3z^3 \div 51xy^2z^3$ (v) $12a^8b^8 \div (-6a^6b^4)$

2. Divide the given polynomial by the given monomial.

(i) $(5x^2 - 6x) \div 3x$ (ii) $(3y^8 - 4y^6 + 5y^4) \div y^4$

(iii) $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$ (iv) $(x^3 + 2x^2 + 3x) \div 2x$

(v) $(p^3q^6 - p^6q^3) \div p^3q^3$

3. Work out the following divisions.

(i) $(10x - 25) \div 5$ (ii) $(10x - 25) \div (2x - 5)$

(iii) $10y(6y + 21) \div 5(2y + 7)$ (iv) $9x^2y^2(3z - 24) \div 27xy(z - 8)$

(v) $96abc(3a - 12)(5b - 30) \div 144(a - 4)(b - 6)$

4. Divide as directed.

(i) $5(2x + 1)(3x + 5) \div (2x + 1)$ (ii) $26xy(x + 5)(y - 4) \div 13x(y - 4)$

(iii) $52pqr(p + q)(q + r)(r + p) \div 104pq(q + r)(r + p)$

(iv) $20(y + 4)(y^2 + 5y + 3) \div 5(y + 4)$ (v) $x(x + 1)(x + 2)(x + 3) \div x(x + 1)$

5. Factorise the expressions and divide them as directed.

(i) $(y^2 + 7y + 10) \div (y + 5)$ (ii) $(m^2 - 14m - 32) \div (m + 2)$

(iii) $(5p^2 - 25p + 20) \div (p - 1)$ (iv) $4yz(z^2 + 6z - 16) \div 2y(z + 8)$

(v) $5pq(p^2 - q^2) \div 2p(p + q)$

(vi) $12xy(9x^2 - 16y^2) \div 4xy(3x + 4y)$ (vii) $39y^3(50y^2 - 98) \div 26y^2(5y + 7)$

WHAT HAVE WE DISCUSSED?

1. When we factorise an expression, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions.

2. An irreducible factor is a factor which cannot be expressed further as a product of factors.

3. A systematic way of factorising an expression is the common factor method. It consists of three steps: (i) Write each term of the expression as a product of irreducible factors (ii) Look for and separate the common factors and (iii) Combine the remaining factors in each term in accordance with the distributive law.

4. Sometimes, all the terms in a given expression do not have a common factor; but the terms can be grouped in such a way that all the terms in each group have a common factor. When we do this, there emerges a common factor across all the groups leading to the required factorisation of the expression. This is the method of regrouping.

5. In factorisation by regrouping, we should remember that any regrouping (i.e., rearrangement) of the terms in the given expression may not lead to factorisation. We must observe the expression and come out with the desired regrouping by trial and error.

© NCERT not to be republished

6. A number of expressions to be factorised are of the form or can be put into the form : $a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$, $a^2 - b^2$ and $x^2 + (a + b) + ab$. These expressions can be easily factorised using Identities I, II, III and IV.

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$
$$a^2 - b^2 = (a + b)(a - b)$$
$$x^2 + (a + b)x + ab = (x + a)(x + b)$$

7. In expressions which have factors of the type $(x+a)(x+b)$, remember the numerical term gives ab. Its factors, a and b, should be so chosen that their sum, with signs taken care of, is the coefficient of x.

8. We know that in the case of numbers, division is the inverse of multiplication. This idea is applicable also to the division of algebraic expressions.

9. In the case of division of a polynomial by a monomial, we may carry out the division either by dividing each term of the polynomial by the monomial or by the common factor method.

10. In the case of division of a polynomial by a polynomial, we cannot proceed by dividing each term in the dividend polynomial by the divisor polynomial. Instead, we factorise both the polynomials and cancel their common factors.

11. In the case of divisions of algebraic expressions that we studied in this chapter, we have
Dividend = Divisor × Quotient.
In general, however, the relation is
Dividend = Divisor × Quotient + Remainder
Thus, we have considered in the present chapter only those divisions in which the remainder is zero.

© NCERT not to be republished

CHAPTER

13

Introduction to Graphs

0852CH15

13.1 Introduction

Have you seen graphs in the newspapers, television, magazines, books etc.? The purpose of the graph is to show numerical facts in visual form so that they can be understood quickly, easily and clearly. Thus graphs are visual representations of data collected. Data can also be presented in the form of a table; however a graphical presentation is easier to understand. This is true in particular when there is **a trend** or **comparison** to be shown.

We have already seen some types of graphs. Let us quickly recall them here.

13.1.1 A line graph

A **line graph** displays data that changes continuously over periods of time.

When Renu fell sick, her doctor maintained a record of her body temperature, taken every four hours. It was in the form of a graph (shown in Fig 13.1 and Fig 13.2).

We may call this a "time-temperature graph".

It is a pictorial representation of the following data, given in tabular form.

Time	6 a.m.	10 a.m.	2 p.m.	6 p.m.
Temperature(°C)	37	40	38	35

The horizontal line (usually called the x-axis) shows the timings at which the temperatures were recorded. What are labelled on the vertical line (usually called the y-axis)?

© NCERT not to be republished

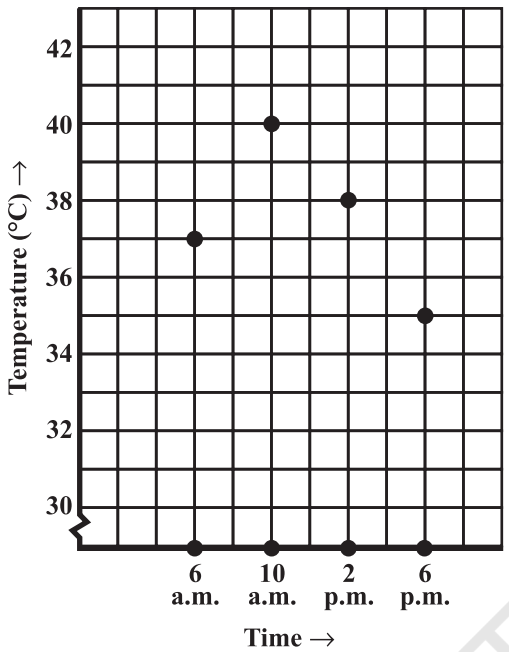


Fig 13.1
Each piece of data is shown by a point on the square grid.

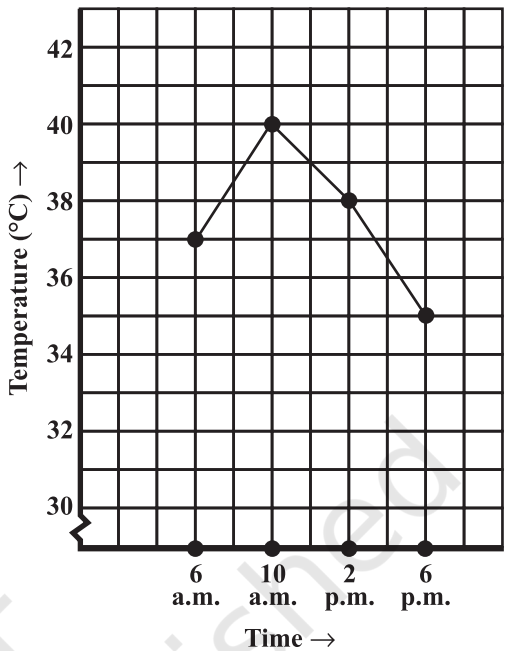


Fig 13.2
The points are then connected by line segments. The result is the **line graph**.

What all does this graph tell you? For example you can see the pattern of temperature; more at 10 a.m. (see Fig 13.3) and then decreasing till 6 p.m. Notice that the temperature increased by 3° C(= 40° C – 37° C) during the period 6 a.m. to 10 a.m.

There was no recording of temperature at 8 a.m., however the graph *suggests* that it was more than 37 °C (How?).

Example 1: (A graph on "performance")

The given graph (Fig 13.3) represents the total runs scored by two batsmen A and B, during each of the ten different matches in the year 2007. Study the graph and answer the following questions.

(i) What information is given on the two axes?
(ii) Which line shows the runs scored by batsman A?
(iii) Were the run scored by them same in any match in 2007? If so, in which match?
(iii) Among the two batsmen, who is steadier? How do you judge it?

Solution:

(i) The horizontal axis (or the x-axis) indicates the matches played during the year 2007. The vertical axis (or the y-axis) shows the total runs scored in each match.

(ii) The dotted line shows the runs scored by Batsman A. (This is already indicated at the top of the graph).

© NCERT not to be republished

(iii) During the 4th match, both have scored the same number of 60 runs. (This is indicated by the point at which both graphs meet).

(iv) Batsman A has one great "peak" but many deep "valleys". He does not appear to be consistent. B, on the other hand has never scored below a total of 40 runs, even though his highest score is only 100 in comparison to 115 of A. Also A has scored a zero in two matches and in a total of 5 matches he has scored less than 40 runs. Since A has a lot of ups and downs, B is a more consistent and reliable batsman.

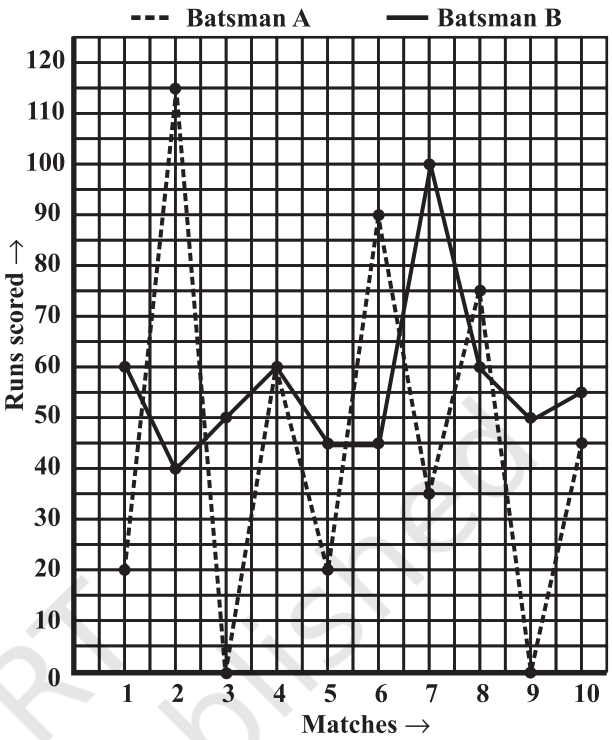


Fig 13.3

Example 2: The given graph (Fig 13.4) describes the distances of a car from a city P at different times when it is travelling from City P to City Q, which are 350 km apart. Study the graph and answer the following:

(i) What information is given on the two axes?

(ii) From where and when did the car begin its journey?

(iii) How far did the car go in the first hour?

(iv) How far did the car go during (i) the 2nd hour? (ii) the 3rd hour?

(v) Was the speed same during the first three hours? How do you know it?

(vi) Did the car stop for some duration at any place? Justify your answer.

(vii) When did the car reach City Q?

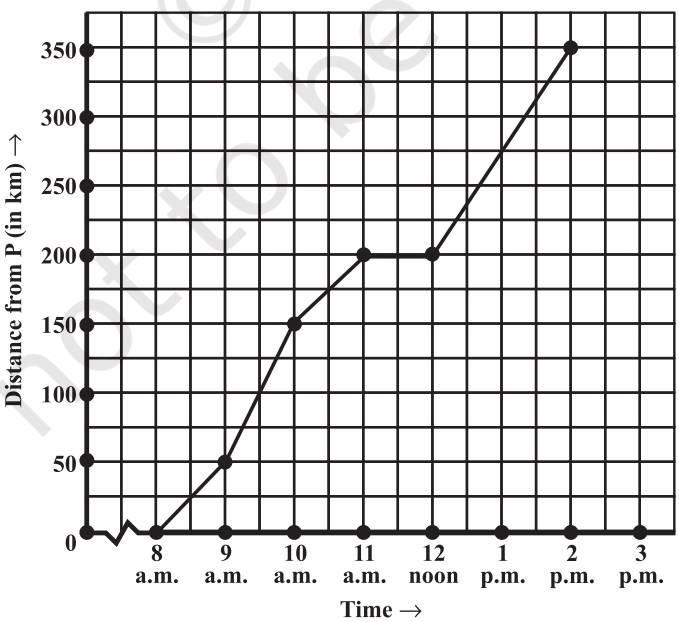


Fig 13.4

© NCERT not to be republished

Solution:

(i) The horizontal (x) axis shows the time. The vertical (y) axis shows the distance of the car from City P.

(ii) The car started from City P at 8 a.m.

(iii) The car travelled 50 km during the first hour. [This can be seen as follows. At 8 a.m. it just started from City P. At 9 a.m. it was at the 50th km (seen from graph). Hence during the one-hour time between 8 a.m. and 9 a.m. the car travelled 50 km].

(iv) The distance covered by the car during

(a) the 2nd hour (i.e., from 9 am to 10 am) is 100 km, (150 – 50).

(b) the 3rd hour (i.e., from 10 am to 11 am) is 50 km (200 – 150).

(v) From the answers to questions (iii) and (iv), we find that the speed of the car was not the same all the time. (In fact the graph illustrates how the speed varied).

(vi) We find that the car was 200 km away from city P when the time was 11 a.m. and also at 12 noon. This shows that the car did not travel during the interval 11 a.m. to 12 noon. The horizontal line segment representing "travel" during this period is illustrative of this fact.

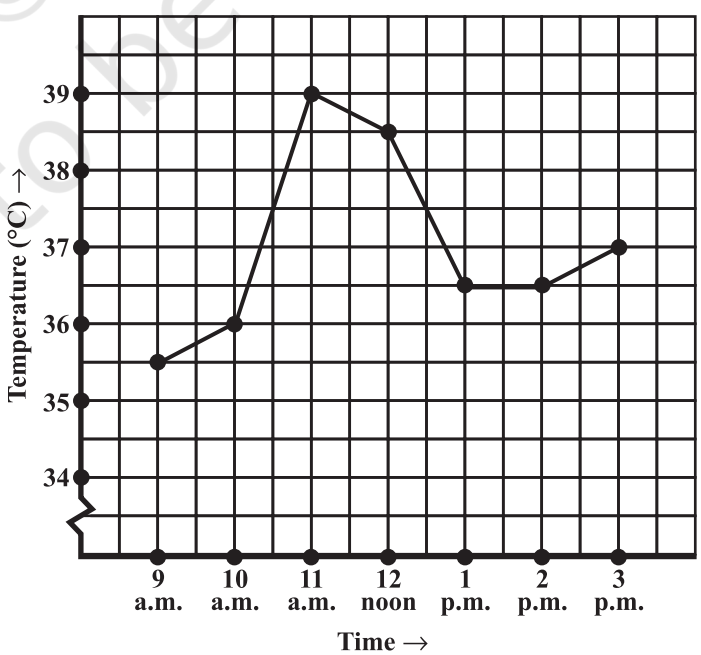
(vii) The car reached City Q at 2 p.m.

EXERCISE 13.1

1. The following graph shows the temperature of a patient in a hospital, recorded every hour.

(a) What was the patient's temperature at 1 p.m. ?

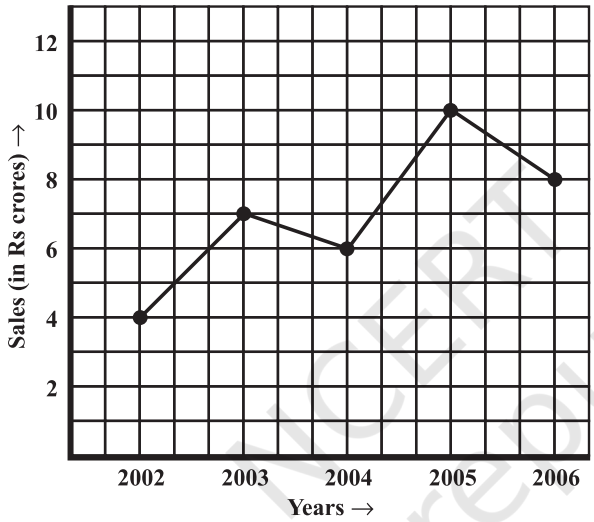
(b) When was the patient's temperature 38.5° C?



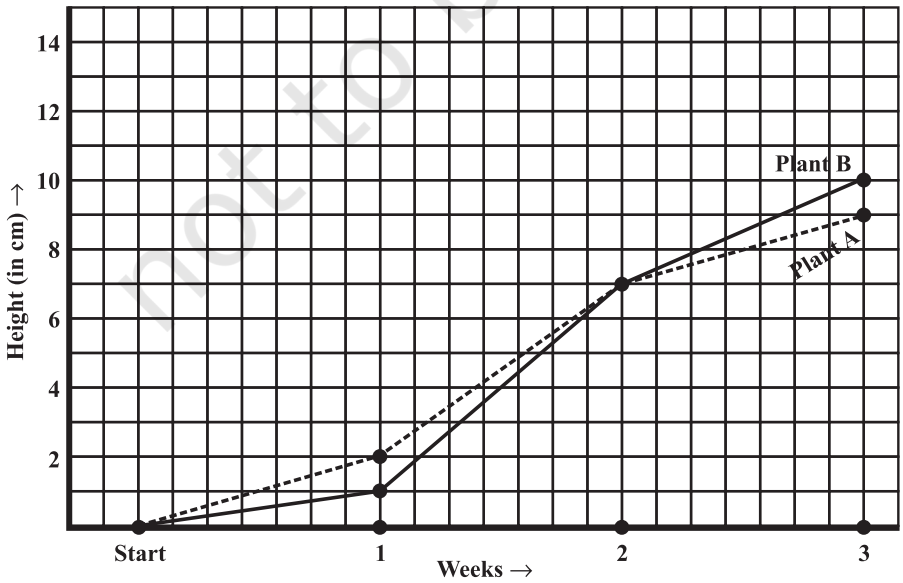
© NCERT not to be republished

(c) The patient's temperature was the same two times during the period given. What were these two times?
(d) What was the temperature at 1.30 p.m.? How did you arrive at your answer?
(e) During which periods did the patients' temperature showed an upward trend?

2. The following line graph shows the yearly sales figures for a manufacturing company.
(a) What were the sales in (i) 2002 (ii) 2006?
(b) What were the sales in (i) 2003 (ii) 2005?
(c) Compute the difference between the sales in 2002 and 2006.
(d) In which year was there the greatest difference between the sales as compared to its previous year?



3. For an experiment in Botany, two different plants, plant A and plant B were grown under similar laboratory conditions. Their heights were measured at the end of each week for 3 weeks. The results are shown by the following graph.

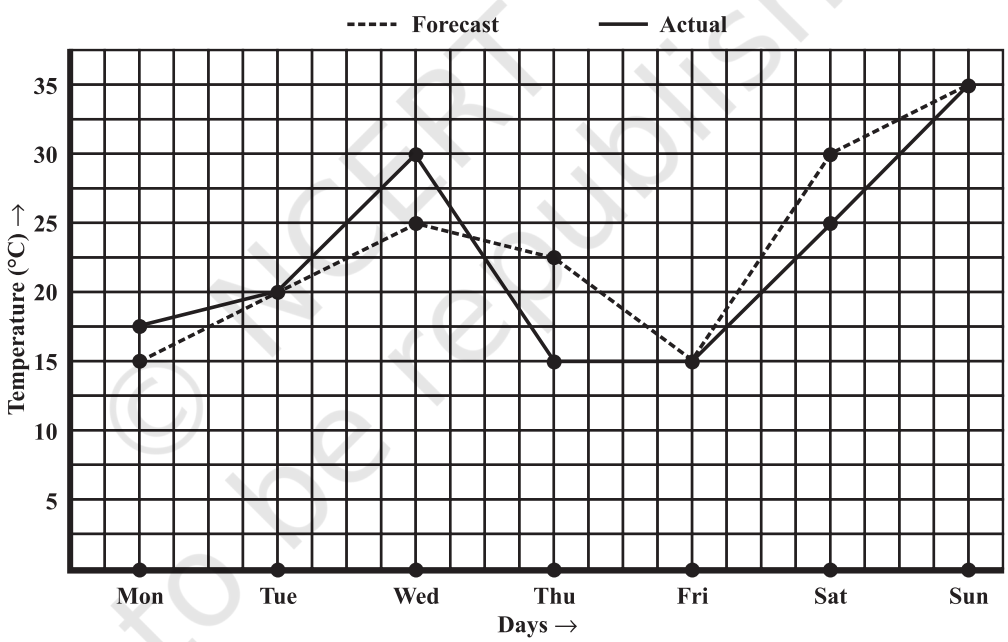


© NCERT not to be republished

(a) How high was Plant A after (i) 2 weeks (ii) 3 weeks?
(b) How high was Plant B after (i) 2 weeks (ii) 3 weeks?
(c) How much did Plant A grow during the 3rd week?
(d) How much did Plant B grow from the end of the 2nd week to the end of the 3rd week?
(e) During which week did Plant A grow most?
(f) During which week did Plant B grow least?
(g) Were the two plants of the same height during any week shown here? Specify.

4. The following graph shows the temperature forecast and the actual temperature for each day of a week.

(a) On which days was the forecast temperature the same as the actual temperature?
(b) What was the maximum forecast temperature during the week?
(c) What was the minimum actual temperature during the week?
(d) On which day did the actual temperature differ the most from the forecast temperature?



5. Use the tables below to draw linear graphs.

(a) The number of days a hill side city received snow in different years.

Year	2003	2004	2005	2006
Days	8	10	5	12

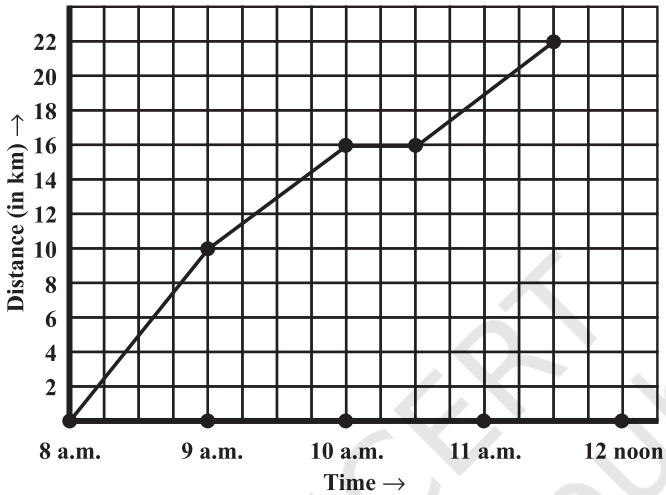
(b) Population (in thousands) of men and women in a village in different years.

Year	2003	2004	2005	2006	2007
Number of Men	12	12.5	13	13.2	13.5
Number of Women	11.3	11.9	13	13.6	12.8

© NCERT not to be republished

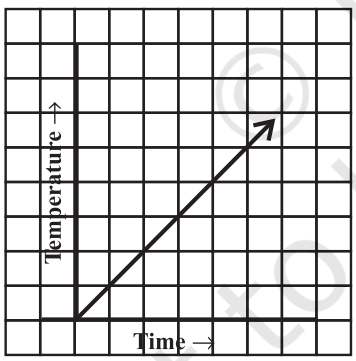
6. A courier-person cycles from a town to a neighbouring suburban area to deliver a parcel to a merchant. His distance from the town at different times is shown by the following graph.

(a) What is the scale taken for the time axis?
(b) How much time did the person take for the travel?
(c) How far is the place of the merchant from the town?
(d) Did the person stop on his way? Explain.
(e) During which period did he ride fastest?

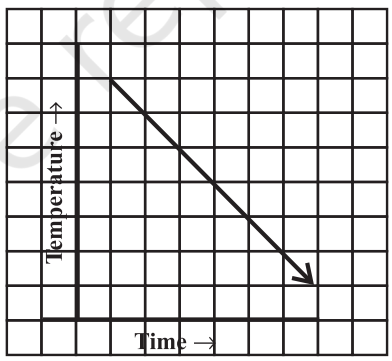


7. Can there be a time-temperature graph as follows? Justify your answer.

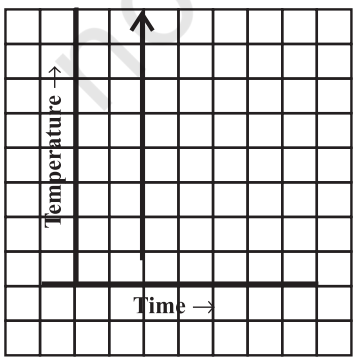
(i)



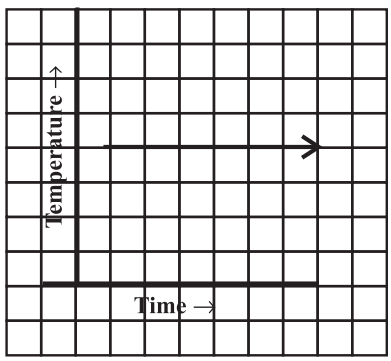
(ii)



(iii)



(iv)



© NCERT not to be republished

13.2 Some Applications

In everyday life, you might have observed that the more you use a facility, the more you pay for it. If more electricity is consumed, the bill is bound to be high. If less electricity is used, then the bill will be easily manageable. This is an instance where one quantity affects another. Amount of electric bill depends on the quantity of electricity used. We say that the quantity of electricity is an **independent variable** (or sometimes **control variable**) and the amount of electric bill is **the dependent variable**. The relation between such variables can be shown through a graph.

THINK, DISCUSS AND WRITE

The number of litres of petrol you buy to fill a car's petrol tank will decide the amount you have to pay. Which is the independent variable here? Think about it.

Example 3: **(Quantity and Cost)**

The following table gives the quantity of petrol and its cost.

No. of Litres of petrol	10	15	20	25
Cost of petrol in ₹	500	750	1000	1250

Plot a graph to show the data.

Solution: (i) Let us take a suitable scale on both the axes (Fig 13.5).

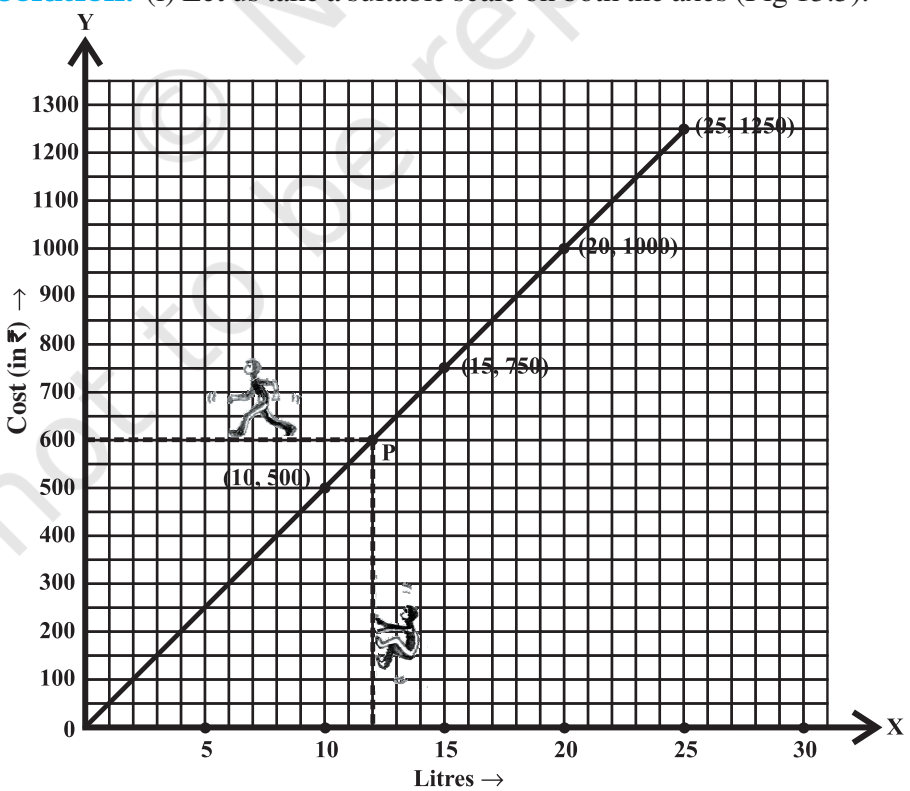


Fig 13.5

© NCERT not to be republished

(ii) Mark number of litres along the horizontal axis.

(iii) Mark cost of petrol along the vertical axis.

(iv) Plot the points: (10,500), (15,750), (20,1000), (25,1250).

(v) Join the points.

We find that the graph is a line. (It is a linear graph). Why does this graph pass through the origin? Think about it.

This graph can help us to estimate a few things. Suppose we want to find the amount needed to buy 12 litres of petrol. Locate 12 on the horizontal axis.

Follow the vertical line through 12 till you meet the graph at P (say).

From P you take a horizontal line to meet the vertical axis. This meeting point provides the answer.

This is the graph of a situation in which two quantities, are in direct variation. (How ?).

In such situations, the graphs will always be linear.

TRY THESE

In the above example, use the graph to find how much petrol can be purchased for ₹ 800.

Example 4: (Principal and Simple Interest)

A bank gives 10% Simple Interest (S.I.) on deposits by senior citizens. Draw a graph to illustrate the relation between the sum deposited and simple interest earned. Find from your graph

(a) the annual interest obtainable for an investment of ₹ 250.

(b) the investment one has to make to get an annual simple interest of ₹ 70.

Solution:

Sum deposited	Simple interest for a year
₹ 100	₹ $\frac{100\times1\times10}{100}$ = ₹ 10
₹ 200	₹ $\frac{200\times1\times10}{100}$ = ₹ 20
₹ 300	₹ $\frac{300\times1\times10}{100}$ = ₹ 30
₹ 500	₹ $\frac{500\times1\times10}{100}$ = ₹ 50
₹ 1000	₹ 100

Steps to follow:

1. Find the quantities to be plotted as Deposit and SI.
2. Decide the quantities to be taken on x-axis and on y-axis.
3. Choose a scale.
4. Plot points.
5. Join the points.

© NCERT not to be republished

We get a table of values.

Deposit (in ₹)	100	200	300	500	1000
Annual S.I. (in ₹)	10	20	30	50	100

(i) Scale : 1 unit = ₹ 100 on horizontal axis; 1 unit = ₹ 10 on vertical axis.

(ii) Mark Deposits along horizontal axis.

(iii) Mark Simple Interest along vertical axis.

(iv) Plot the points : (100,10), (200, 20), (300, 30), (500,50) etc.

(v) Join the points. We get a graph that is a line (Fig 13.6).

(a) Corresponding to ₹ 250 on horizontal axis, we get the interest to be ₹ 25 on vertical axis.

(b) Corresponding to ₹ 70 on the vertical axis, we get the sum to be ₹ 700 on the horizontal axis.

TRY THESE

Is Example 4, a case of direct variation?

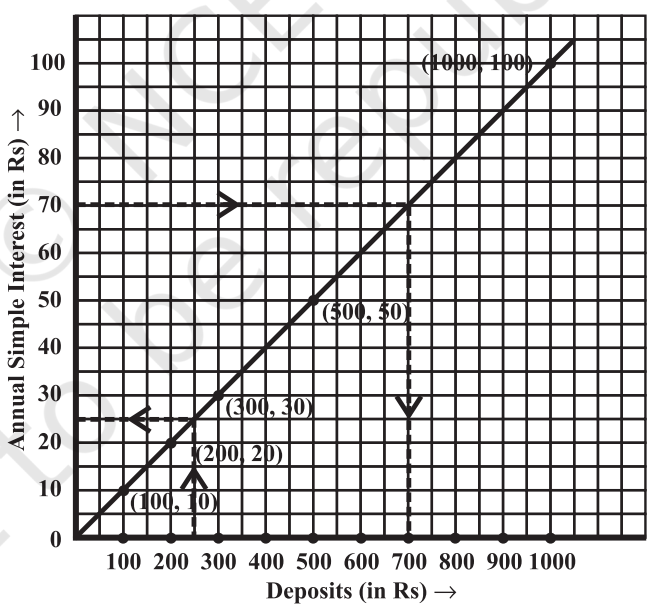


Fig 13.6

Example 5: (Time and Distance)

Ajit can ride a scooter constantly at a speed of 30 kms/hour. Draw a time-distance graph for this situation. Use it to find

(i) the time taken by Ajit to ride 75 km. (ii) the distance covered by Ajit in $3\frac{1}{2}$ hours.

© NCERT not to be republished

Solution:

Hours of ride	Distance covered
1 hour	30 km
2 hours	2 × 30 km = 60 km
3 hours	3 × 30 km = 90 km
4 hours	4 × 30 km = 120 km and so on.

We get a table of values.

Time (in hours)	1	2	3	4
Distance covered (in km)	30	60	90	120

(i) Scale: (Fig 13.7)

Horizontal: 2 units = 1 hour

Vertical: 1 unit = 10 km

(ii) Mark time on horizontal axis.

(iii) Mark distance on vertical axis.

(iv) Plot the points: (1, 30), (2, 60), (3, 90), (4, 120).

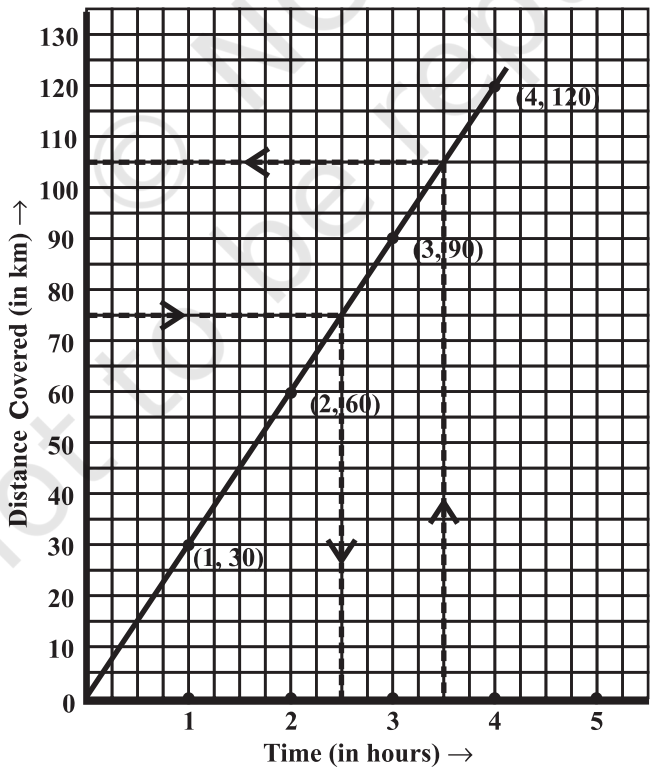


Fig 13.7

© NCERT not to be republished

(v) Join the points. We get a linear graph.

(a) Corresponding to 75 km on the vertical axis, we get the time to be 2.5 hours on the horizontal axis. Thus 2.5 hours are needed to cover 75 km.

(b) Corresponding to $3\frac{1}{2}$ hours on the horizontal axis, the distance covered is 105 km on the vertical axis.

EXERCISE 13.2

1. Draw the graphs for the following tables of values, with suitable scales on the axes.

(a) Cost of apples

Number of apples	1	2	3	4	5
Cost (in ₹)	5	10	15	20	25

(b) Distance travelled by a car

Time (in hours)	6 a.m.	7 a.m.	8 a.m.	9 a.m.
Distances (in km)	40	80	120	160

(i) How much distance did the car cover during the period 7.30 a.m. to 8 a.m?

(ii) What was the time when the car had covered a distance of 100 km since it's start?

(c) Interest on deposits for a year.

Deposit (in ₹)	1000	2000	3000	4000	5000
Simple Interest (in ₹)	80	160	240	320	400

(i) Does the graph pass through the origin?

(ii) Use the graph to find the interest on ₹ 2500 for a year.

(iii) To get an interest of ₹ 280 per year, how much money should be deposited?

2. Draw a graph for the following.

(i)

Side of square (in cm)	2	3	3.5	5	6
Perimeter (in cm)	8	12	14	20	24

Is it a linear graph?

© NCERT not to be republished

(ii)

Side of square (in cm)	2	3	4	5	6
Area (in cm^2)	4	9	16	25	36

Is it a linear graph?

WHAT HAVE WE DISCUSSED?

1. Graphical presentation of data is easier to understand.
2. A **line graph** displays data that changes continuously over periods of time.
3. A line graph which is a whole unbroken line is called a **linear graph**.
4. For fixing a point on the graph sheet we need, ***x*-coordinate** and ***y*-coordinate**.
5. The relation between **dependent variable** and **independent variable** is shown through a graph.

© NCERT not to be republished

NOTES

© NCERT not to be republished